DBE

MATHEMATICS

PAST PAPER BOOKLET 2020

Past Paper Questions By Topic

PAPER 2

AngloAmerican PLATINUM

XLEducation

PAPER 2 QUESTIONS

Mathematics
Past Paper Revision By Topic

5. INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ *area* $\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS

EXAMINATION GUIDELINES

SENIOR CERTIFICATE (SC)

GRADE 12

2015

These guidelines consist of 16 pages.

CONTENTS **Page**

1. INTRODUCTION

The Curriculum and Assessment Policy Statement (CAPS) for Mathematics outlines the nature and purpose of the subject Mathematics. This guides the philosophy underlying the teaching and assessment of the subject in Grade 12.

The purpose of these Examination Guidelines is to provide clarity on the depth and scope of the content to be assessed in the Grade 12 Senior Certificate (SC) Examination in Mathematics.

These Examination Guidelines should be read in conjunction with:

- A resumé of subjects for the Senior Certificate
- Curriculum and Assessment Policy Statements for all approved subjects

2. ASSESSMENT IN GRADE 12

All candidates will write two question papers as prescribed.

2.1 Format of question papers for Grade 12

Paper	Topics	Duration	Total
1	Patterns and sequences Finance, growth and decay Functions and graphs Algebra, equations and inequalities Differential Calculus Probability	3 hours	150
2	Euclidean Geometry Analytical Geometry Statistics and regression Trigonometry	3 hours	150

Questions in both Papers 1 and 2 will assess performance at different cognitive levels with an emphasis on process skills, critical thinking, scientific reasoning and strategies to investigate and solve problems in a variety of contexts.

An Information Sheet is included on p. 15.

2.2 Weighting of cognitive levels

Papers 1 and 2 will include questions across four cognitive levels. The distribution of cognitive levels in the papers is given below.

Cognitive level	Description of skills to be demonstrated	Weighting	Approximate number of marks in a 150-mark paper
Knowledge	• Recall • Identification of correct formula on the information sheet (no changing of the subject) • Use of mathematical facts • Appropriate use of mathematical vocabulary • Algorithms • Estimation and appropriate rounding of numbers	20%	30 marks
Routine Procedures	• Proofs of prescribed theorems and derivation of formulae • Perform well-known procedures • Simple applications and calculations which might involve few steps • Derivation from given information may be involved • Identification and use (after changing the subject) of correct formula • Generally similar to those encountered in class	35%	52–53 marks
Complex Procedures	• Problems involve complex calculations and/or higher order reasoning • There is often not an obvious route to the solution • Problems need not be based on a real world context • Could involve making significant connections between different representations • Require conceptual understanding • Candidates are expected to solve problems by integrating different topics.	30%	45 marks
Problem Solving	• Non-routine problems (which are not necessarily difficult) • Problems are mainly unfamiliar • Higher order reasoning and processes are involved • Might require the ability to break the problem down into its constituent parts • Interpreting and extrapolating from solutions obtained by solving problems based in unfamiliar contexts.	15%	22–23 marks

3. ELABORATION OF CONTENT/TOPICS

The purpose of the clarification of the topics is to give guidance to the teacher in terms of depth of content necessary for examination purposes. Integration of topics is encouraged as candidates should understand Mathematics as a holistic discipline. Thus questions integrating various topics can be asked.

FUNCTIONS

1. Candidates must be able to use and interpret functional notation. In the teaching process candidates must be able to understand how $f(x)$ has been transformed to generate $f(-x)$, $-f(x)$, $f(x+a)$, $f(x)+a$, $af(x)$ and $x=f(y)$ where $a \in R$.
2. Trigonometric functions will ONLY be examined in Paper 2.

NUMBER PATTERNS, SEQUENCES AND SERIES

1. The sequence of first differences of a quadratic number pattern is linear. Therefore, knowledge of linear patterns can be tested in the context of quadratic number patterns.
2. Recursive patterns will not be examined explicitly.
3. Links must be clearly established between patterns done in earlier grades.

FINANCE, GROWTH AND DECAY

1. Understand the difference between nominal and effective interest rates and convert fluently between them for the following compounding periods: monthly, quarterly and half-yearly or semi-annually.
2. With the exception of calculating i in the F_v and P_v formulae, candidates are expected to calculate the value of any of the other variables.
3. Pyramid schemes will not be examined in the examination.

ALGEBRA

1. Solving quadratic equations by completing the square will not be examined.
2. Solving quadratic equations using the substitution method (*k*-method) is examinable.
3. Equations involving surds that lead to a quadratic equation are examinable.
4. Solution of non-quadratic inequalities should be seen in the context of functions.
5. Nature of the roots will be tested intuitively with the solution of quadratic equations and in all the prescribed functions.

DIFFERENTIAL CALCULUS

1. The following notations for differentiation can be used: $f'(x)$, D_x, $\frac{dy}{dx}$ or y'.
2. In respect of cubic functions, candidates are expected to be able to:
   - Determine the equation of a cubic function from a given graph.

- Discuss the nature of stationary points including local maximum, local minimum and points of inflection.
- Apply knowledge of transformations on a given function to obtain its image.

3. Candidates are expected to be able to draw and interpret the graph of the derivative of a function.
4. Surface area and volume will be examined in the context of optimisation.
5. Candidates must know the formulae for the surface area and volume of the right prisms. These formulae will not be provided on the formula sheet
6. If the optimisation question is based on the surface area and/or volume of the cone, sphere and/or pyramid, a list of the relevant formulae will be provided in that question. Candidates will be expected to select the correct formula from this list.

PROBABILITY

1. Dependent events are examinable but conditional probabilities are not part of the syllabus.
2. Dependent events in which an object is not replaced is examinable.
3. Questions that require the candidate to count the different number of ways that objects may be arranged in a circle and/or the use of combinations are not in the spirit of the curriculum.
4. In respect of word arrangements, letters that are repeated in the word can be treated as the same (indistinguishable) or different (distinguishable). The question will be specific in this regard.

EUCLIDEAN GEOMETRY & MEASUREMENT

1. Measurement can be tested in the context of optimisation in calculus.
2. Composite shapes could be formed by combining a maximum of TWO of the stated shapes.
3. The following proofs of theorems are examinable:
   - The line drawn from the centre of a circle perpendicular to a chord bisects the chord;
   - The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre);
   - The opposite angles of a cyclic quadrilateral are supplementary;
   - The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment;
   - A line drawn parallel to one side of a triangle divides the other two sides proportionally;
   - Equiangular triangles are similar.
4. Corollaries derived from the theorems and axioms are necessary in solving riders:
   - Angles in a semi-circle
   - Equal chords subtend equal angles at the circumference
   - Equal chords subtend equal angles at the centre
   - In equal circles, equal chords subtend equal angles at the circumference
   - In equal circles, equal chords subtend equal angles at the centre.
   - The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle of the quadrilateral.
   - If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.
   - Tangents drawn from a common point outside the circle are equal in length.

5. The theory of quadrilaterals will be integrated into questions in the examination.
6. Concurrency theory is excluded.

TRIGONOMETRY

1. The reciprocal ratios cosec θ, sec θ and cot θ can be used by candidates in the answering of problems but will not be explicitly tested.
2. The focus of trigonometric graphs is on the relationships, simplification and determining points of intersection by solving equations, although characteristics of the graphs should not be excluded.

ANALYTICAL GEOMETRY

1. Prove the properties of polygons by using analytical methods.
2. The concept of collinearity must be understood.
3. Candidates are expected to be able to integrate Euclidean Geometry axioms and theorems into Analytical Geometry problems.
4. The length of a tangent from a point outside the circle should be calculated.
5. Concepts involved with concurrency will not be examined.

STATISTICS

1. Candidates should be encouraged to use the calculator to calculate standard deviation, variance and the equation of the least squares regression line.
2. The interpretation of standard deviation in terms of normal distribution is not examinable.
3. Candidates are expected to identify outliers intuitively in both the scatter plot as well as the box and whisker diagram.

   In the case of the box and whisker diagram, observations that lie outside the interval (lower quartile – 1,5 IQR ; upper quartile + 1,5 IQR) are considered to be outliers. However, candidates will not be penalised if they did not make use of this formula in identifying outliers.

4. ACCEPTABLE REASONS: EUCLIDEAN GEOMETRY

In order to have some kind of uniformity, the use of the following shortened versions of the theorem statements is encouraged.

4.1 ACCEPTABLE REASONS: EUCLIDEAN GEOMETRY (ENGLISH)

THEOREM STATEMENT	ACCEPTABLE REASON(S)
LINES	
The adjacent angles on a straight line are supplementary.	∠s on a str line
If the adjacent angles are supplementary, the outer arms of these angles form a straight line.	adj ∠s supp
The adjacent angles in a revolution add up to 360°.	∠s round a pt **OR** ∠s in a rev
Vertically opposite angles are equal.	vert opp ∠s =
If AB || CD, then the alternate angles are equal.	alt ∠s; AB || CD
If AB || CD, then the corresponding angles are equal.	corresp ∠s; AB || CD
If AB || CD, then the co-interior angles are supplementary.	co-int ∠s; AB || CD
If the alternate angles between two lines are equal, then the lines are parallel.	alt ∠s =
If the corresponding angles between two lines are equal, then the lines are parallel.	corresp ∠s =
If the cointerior angles between two lines are supplementary, then the lines are parallel.	coint ∠s supp
TRIANGLES	
The interior angles of a triangle are supplementary.	∠ sum in Δ **OR** sum of ∠s in Δ **OR** Int ∠s Δ
The exterior angle of a triangle is equal to the sum of the interior opposite angles.	ext ∠ of Δ
The angles opposite the equal sides in an isosceles triangle are equal.	∠s opp equal sides
The sides opposite the equal angles in an isosceles triangle are equal.	sides opp equal ∠s
In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.	Pythagoras **OR** Theorem of Pythagoras
If the square of the longest side in a triangle is equal to the sum of the squares of the other two sides then the triangle is right-angled.	Converse Pythagoras **OR** Converse Theorem of Pythagoras
If three sides of one triangle are respectively equal to three sides of another triangle, the triangles are congruent.	SSS
If two sides and an included angle of one triangle are respectively equal to two sides and an included angle of another triangle, the triangles are congruent.	SAS **OR** S∠S
If two angles and one side of one triangle are respectively equal to two angles and the corresponding side in another triangle, the triangles are congruent.	AAS **OR** ∠∠S
If in two right angled triangles, the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other, the triangles are congruent	RHS **OR** 90°HS

THEOREM STATEMENT	ACCEPTABLE REASON(S)
The line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half the length of the third side	Midpt Theorem
The line drawn from the midpoint of one side of a triangle, parallel to another side, bisects the third side.	line through midpt ‖ to 2^{nd} side
A line drawn parallel to one side of a triangle divides the other two sides proportionally.	line ‖ one side of Δ **OR** prop theorem; name ‖ lines
If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side.	line divides two sides of Δ in prop
If two triangles are equiangular, then the corresponding sides are in proportion (and consequently the triangles are similar).	‖‖ Δs **OR** equiangular Δs
If the corresponding sides of two triangles are proportional, then the triangles are equiangular (and consequently the triangles are similar).	Sides of Δ in prop
If triangles (or parallelograms) are on the same base (or on bases of equal length) and between the same parallel lines, then the triangles (or parallelograms) have equal areas.	same base; same height **OR** equal bases; equal height
CIRCLES	
The tangent to a circle is perpendicular to the radius/diameter of the circle at the point of contact.	tan ⊥ radius tan ⊥ diameter
If a line is drawn perpendicular to a radius/diameter at the point where the radius/diameter meets the circle, then the line is a tangent to the circle.	line ⊥ radius **OR** converse tan ⊥ radius **OR** converse tan ⊥ diameter
The line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord.	line from centre to midpt of chord
The line drawn from the centre of a circle perpendicular to a chord bisects the chord.	line from centre ⊥ to chord
The perpendicular bisector of a chord passes through the centre of the circle;	perp bisector of chord
The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre)	∠ at centre = 2 ×∠ at circumference
The angle subtended by the diameter at the circumference of the circle is 90°.	∠s in semi circle **OR** diameter subtends right angle **OR** ∠ in $\frac{1}{2}$ ⊙
If the angle subtended by a chord at the circumference of the circle is 90°, then the chord is a diameter.	chord subtends 90° **OR** converse ∠s in semi circle
Angles subtended by a chord of the circle, on the same side of the chord, are equal	∠s in the same seg
If a line segment joining two points subtends equal angles at two points on the same side of the line segment, then the four points are concyclic.	line subtends equal ∠s **OR** converse ∠s in the same seg
Equal chords subtend equal angles at the circumference of the circle.	equal chords; equal ∠s
Equal chords subtend equal angles at the centre of the circle.	equal chords; equal ∠s

THEOREM STATEMENT	ACCEPTABLE REASON(S)
Equal chords in equal circles subtend equal angles at the circumference of the circles.	equal circles; equal chords; equal ∠s
Equal chords in equal circles subtend equal angles at the centre of the circles.	equal circles; equal chords; equal ∠s
The opposite angles of a cyclic quadrilateral are supplementary	opp ∠s of cyclic quad
If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.	opp ∠s quad supp **OR** converse opp ∠s of cyclic quad
The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.	ext ∠ of cyclic quad
If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.	ext ∠ = int opp ∠ **OR** converse ext ∠ of cyclic quad
Two tangents drawn to a circle from the same point outside the circle are equal in length	Tans from common pt **OR** Tans from same pt
The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.	tan chord theorem
If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle.	converse tan chord theorem **OR** ∠ between line and chord
QUADRILATERALS	
The interior angles of a quadrilateral add up to 360°.	sum of ∠s in quad
The opposite sides of a parallelogram are parallel.	opp sides of ||m
If the opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.	opp sides of quad are ||
The opposite sides of a parallelogram are equal in length.	opp sides of ||m
If the opposite sides of a quadrilateral are equal , then the quadrilateral is a parallelogram.	opp sides of quad are = **OR** converse opp sides of a parm
The opposite angles of a parallelogram are equal.	opp ∠s of ||m
If the opposite angles of a quadrilateral are equal then the quadrilateral is a parallelogram.	opp ∠s of quad are = **OR** converse opp angles of a parm
The diagonals of a parallelogram bisect each other.	diag of ||m
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.	diags of quad bisect each other **OR** converse diags of a parm
If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.	pair of opp sides = and ||
The diagonals of a parallelogram bisect its area.	diag bisect area of ||m
The diagonals of a rhombus bisect at right angles.	diags of rhombus
The diagonals of a rhombus bisect the interior angles.	diags of rhombus
All four sides of a rhombus are equal in length.	sides of rhombus
All four sides of a square are equal in length.	sides of square
The diagonals of a rectangle are equal in length.	diags of rect
The diagonals of a kite intersect at right-angles.	diags of kite
A diagonal of a kite bisects the other diagonal.	diag of kite
A diagonal of a kite bisects the opposite angles	diag of kite

7. CONCLUSION

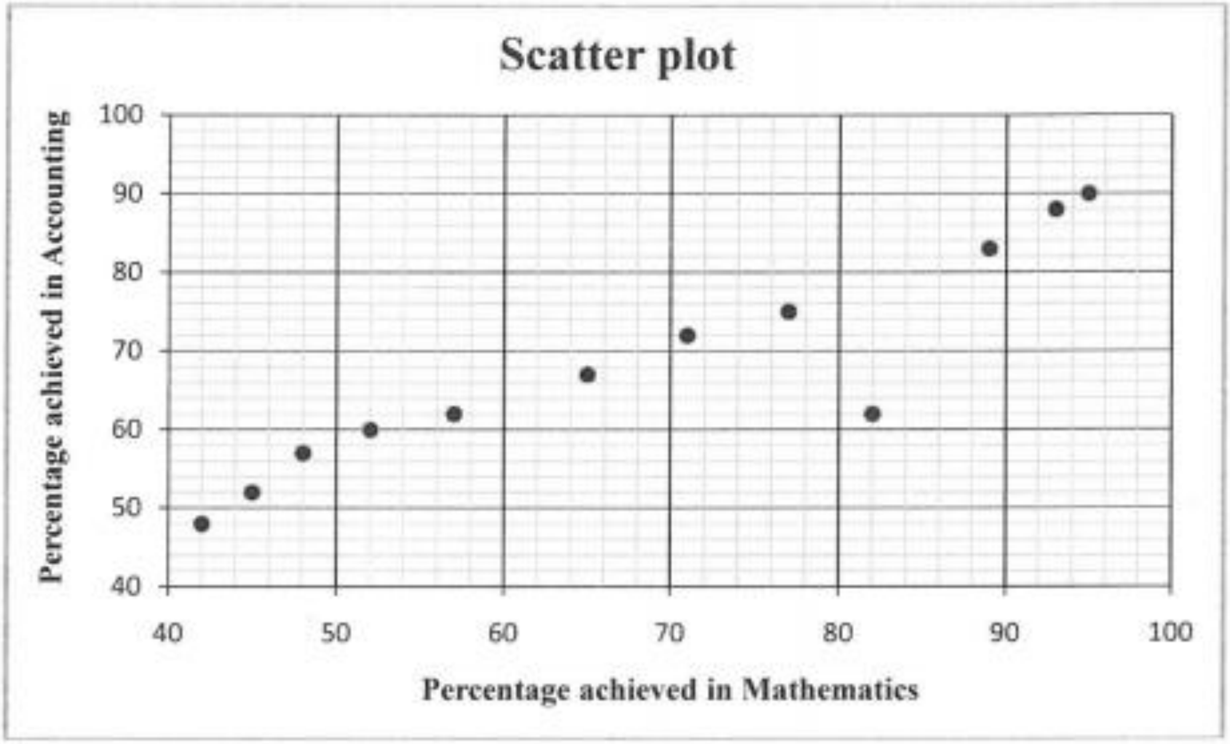
This Examination Guidelines document is meant to articulate the assessment aspirations espoused in the CAPS document. It is therefore not a substitute for the CAPS document which educators should teach to.

Qualitative curriculum coverage as enunciated in the CAPS cannot be over-emphasised.

Question 1 **November 2014**

At a certain school, only 12 candidates take Mathematics and Accounting. The marks, as a percentage, scored by these candidates in the preparatory examinations for Mathematics and Accounting, are shown in the table and scatter plot below.

Mathematics	52	82	93	95	71	65	77	42	89	48	45	57
Accounting	60	62	88	90	72	67	75	48	83	57	52	62



1.1 Calculate the mean percentage of the Mathematics data. (2)

1.2 Calculate the standard deviation of the Mathematics data. (1)

1.3 Determine the number of candidates whose percentages in Mathematics lie within ONE standard deviation of the mean. (3)

1.4 Calculate an equation for the least squares regression line (line of best fit) for the data. (3)

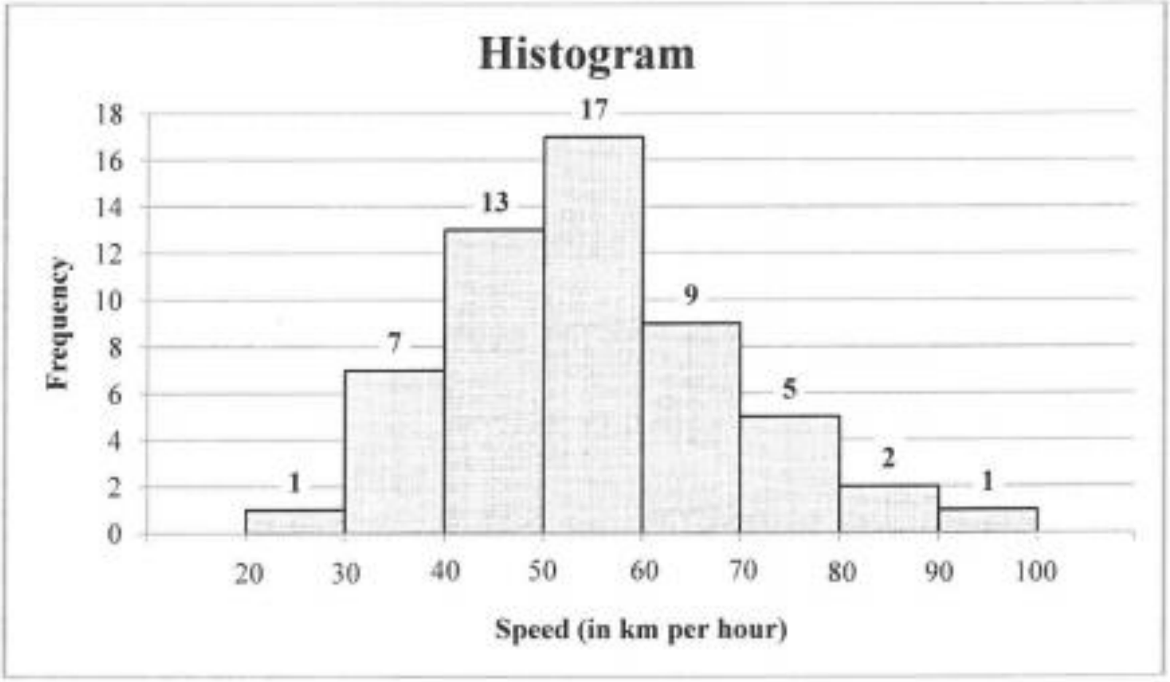
1.5 If a candidate from this group scored 60% in the Mathematics examination but was absent for the Accounting examination, predict the percentage that this candidate would have scored in the Accounting examination, using your equation in QUESTION 1.4. (Round off your answer to the NEAREST INTEGER.) (2)

1.6 Use the scatter plot and identify any outlier(s) in the data. (1)

[12]

Question 2 **November 2014**

The speeds of 55 cars passing through a certain section of a road are monitored for one hour. The speed limit on this section of road is 60 km per hour. A histogram is drawn to represent this data.



2.1 Identify the modal class of the data. (1)

2.2 Use the histogram to:

2.2.1 Complete the cumulative frequency column in the table on DIAGRAM SHEET 1 (2)

2.2.2 Draw an ogive (cumulative frequency graph) of the above data on the grid on DIAGRAM SHEET 1 (3)

2.3 The traffic department sends speeding fines to all motorists whose speed exceeds 66 km per hour. Estimate the number of motorists who will receive a speeding fine. (2)

[8]

Question 1 **Feb March 2015**

The table below shows the distances (in kilometres) travelled daily by a sales representative for 21 working days in a certain month.

131	132	140	140	141	144	146
147	149	150	151	159	167	169
169	172	174	175	178	187	189

1.1 Calculate the mean distance travelled by the sales representative. (2)

1.2 Write down the five-number summary for this set of data. (4)

1.3 Use the scaled line on DIAGRAM SHEET 1 to draw a box-and-whisker diagram for this set of data. (2)

1.4 Comment on the skewness of the data. (1)

1.5 Calculate the standard deviation of the distance travelled. (2)

1.6 The sales representative discovered that his odometer was faulty. The actual reading on each of the 21 days was p km more than that which was indicated. Write down, in terms of p (if applicable), the:

1.6.1 Actual mean (1)

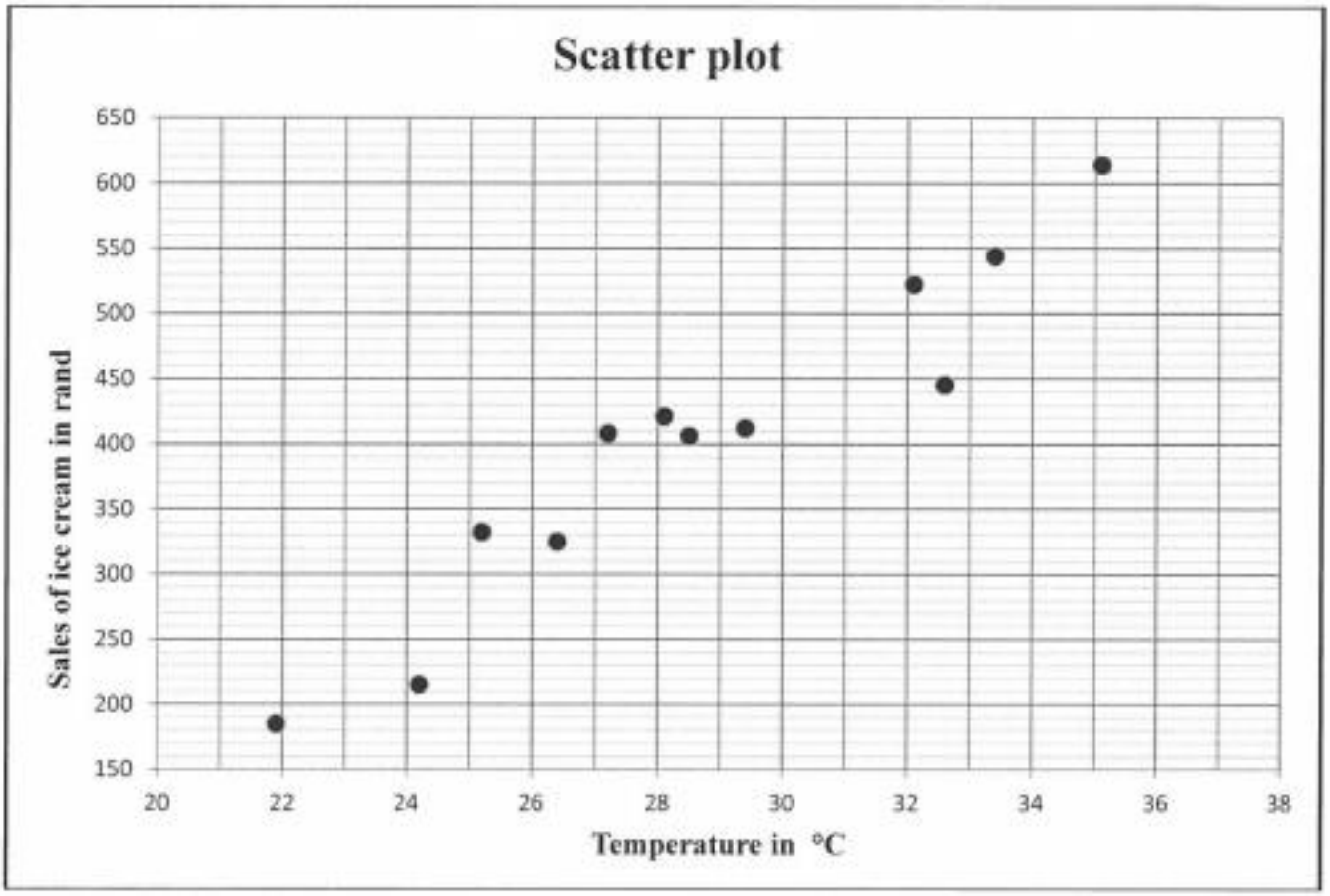
1.6.2 Actual standard deviation (1)

[13]

Question 2 **Feb March 2015**

An ice-cream shop recorded the sales of ice cream, in rand, and the maximum temperature, in °C, for 12 days in a certain month. The data that they collected is represented in the table and scatter plot below.

Temperature in °C	24,2	26,4	21,9	25,2	28,5	32,1	29,4	35,1	33,4	28,1	32,6	27,2
Sales of ice cream in rand	215	325	185	332	406	522	412	614	544	421	445	408



2.1	Describe the influence of temperature on the sales of ice cream in the scatter plot.	(1)
2.2	Give a reason why this trend cannot continue indefinitely.	(1)
2.3	Calculate an equation for the least squares regression line (line of best fit).	(4)
2.4	Calculate the correlation coefficient.	(1)
2.5	Comment on the strength of the relationship between the variables.	(1)
		[8]

Question 1 **November 2015**

The table below shows the total fat (in grams, rounded off to the nearest whole number) and energy (in kilojoules, rounded off to the nearest 100) of 10 items that are sold at a fast-food restaurant.

Fat (in grams)	9	14	25	8	12	31	28	14	29	20
Energy (in kilojoules)	1 100	1 300	2 100	300	1 200	2 400	2 200	1 400	2 600	1 600

1.1 Represent the information above in a scatter plot on the grid provided in the ANSWER BOOK. (3)

1.2 The equation of the least squares regression line is $\hat{y} = 154,60 + 77,13x$.

1.2.1 An item at the restaurant contains 18 grams of fat. Calculate the number of kilojoules of energy that this item will provide. Give your answer rounded off to the nearest 100 kJ. (2)

1.2.2 Draw the least squares regression line on the scatter plot drawn for QUESTION 1.1. (2)

1.3 Identify an outlier in the data set. (1)

1.4 Calculate the value of the correlation coefficient. (2)

1.5 Comment on the strength of the relationship between the fat content and the number of kilojoules of energy. (1)

[11]

Question 2 **November 2015**

A group of 30 learners each randomly rolled two dice once and the sum of the values on the uppermost faces of the dice was recorded. The data is shown in the frequency table below.

Sum of the values on uppermost faces	Frequency
2	0
3	3
4	2
5	4
6	4
7	8
8	3
9	2
10	2
11	1
12	1

2.1 Calculate the mean of the data. (2)

2.2 Determine the median of the data. (2)

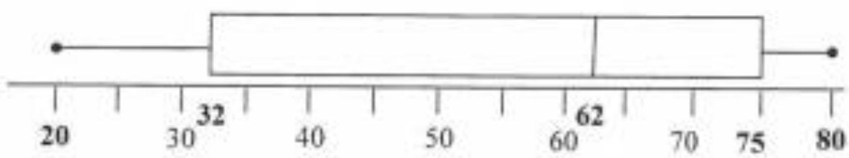
2.3 Determine the standard deviation of the data. (2)

2.4 Determine the number of times that the sum of the recorded values of the dice is within ONE standard deviation from the mean. Show your calculations. (3)

[9]

Question 1 **Feb March 2016**

The box and whisker diagram below shows the marks (out of 80) obtained in a History test by a class of nine learners.



1.1 Comment on the skewness of the data. (1)

1.2 Write down the range of the marks obtained. (2)

1.3 If the learners had to obtain 32 marks to pass the test, estimate the percentage of the class that failed the test. (2)

1.4 In ascending order, the second mark is 28, the third mark 36 and the sixth mark 69. The seventh and eighth marks are the same. The average mark for this test is 54.

	28	36			69			

Fill in the marks of the remaining learners in ascending order. (6)
[11]

Question 2 **Feb March 2016**

A company recorded the number of messages sent by e-mail over a period of 60 working days. The data is shown in the table below.

NUMBER OF MESSAGES	NUMBER OF DAYS
$10 < x \leq 20$	2
$20 < x \leq 30$	8
$30 < x \leq 40$	5
$40 < x \leq 50$	10
$50 < x \leq 60$	12
$60 < x \leq 70$	18
$70 < x \leq 80$	3
$80 < x \leq 90$	2

2.1 Estimate the mean number of messages sent per day, rounded off to TWO decimal places. (3)

2.2 Draw a cumulative frequency graph (ogive) of the data on the grid provided in the ANSWER BOOK. (4)

2.3 Hence, estimate the number of days on which 65 or more messages were sent. (2)
[9]

Question 1 **May June 2016**

On a certain day a tour operator sent 11 tour buses to 11 different destinations. The table below shows the number of passengers on each bus.

8	8	10	12	16	19	20	21	24	25	26

1.1 Calculate the mean number of passengers travelling in a tour bus. (2)

1.2 Write down the five-number summary of the data. (3)

1.3 Draw a box and whisker diagram for the data. Use the number line provided in the ANSWER BOOK. (2)

1.4 Refer to the box and whisker diagram and comment on the skewness of the data set. (1)

1.5 Calculate the standard deviation for this data set. (2)

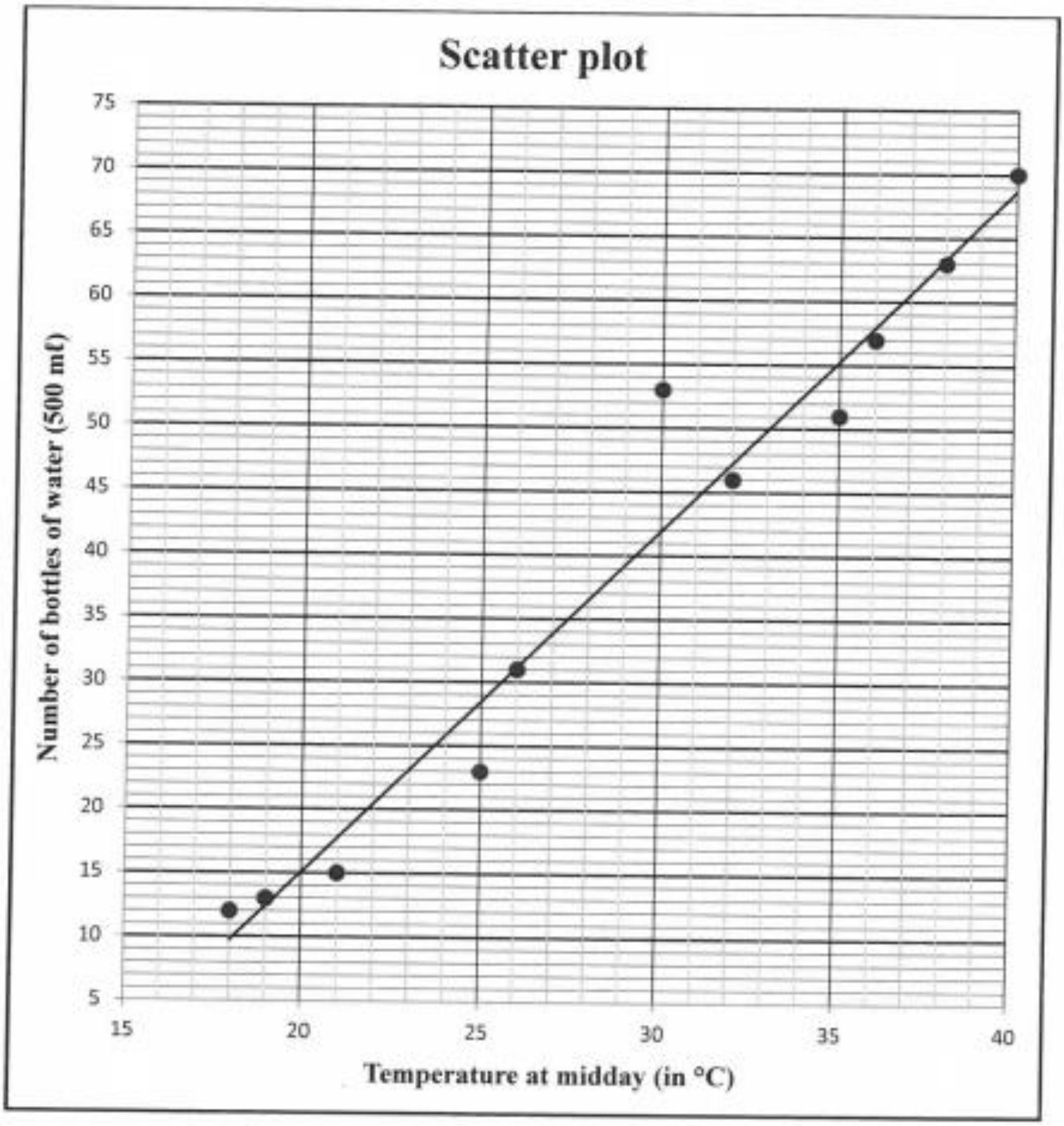
1.6 A tour is regarded as popular if the number of passengers on a tour bus is one standard deviation above the mean. How many destinations were popular on this particular day? (2)

[12]

Question 2 **May June 2016**

On the first school day of each month information is recorded about the temperature at midday (in °C) and the number of 500 mℓ bottles of water that were sold at the tuck shop of a certain school during the lunch break. The data is shown in the table below and represented on the scatter plot. The least squares regression line for this data is drawn on the scatter plot.

Temperature at midday (in °C)	18	21	19	26	32	35	36	40	38	30	25
Number of bottles of water (500 mℓ)	12	15	13	31	46	51	57	70	63	53	23

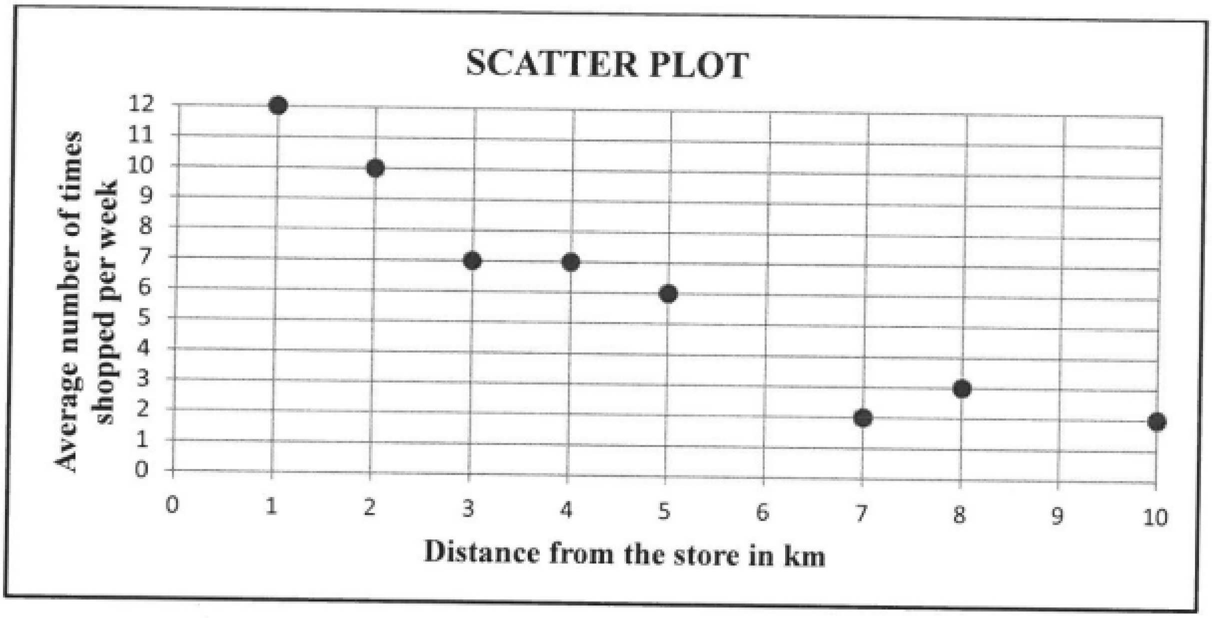


2.1	Identify an outlier in the data.	(1)
2.2	Determine the equation of the least squares regression line.	(3)
2.3	Estimate the number of 500 mℓ bottles of water that will be sold if the temperature is 28 °C at midday.	(2)
2.4	Refer to the scatter plot. Would you say that the relation between the temperature at midday and the number of 500 mℓ bottles of water sold is weak or strong? Motivate your answer.	(2)
2.5	Give a reason why the observed trend for this data cannot continue indefinitely.	(1)
		[9]

Question 1 **November 2016**

A survey was conducted at a local supermarket relating the distance that shoppers lived from the store to the average number of times they shopped at the store in a week. The results are shown in the table below.

Distance from the store in km	1	2	3	4	5	7	8	10
Average number of times shopped per week	12	10	7	7	6	2	3	2



1.1 Use the scatter plot to comment on the strength of the relationship between the distance a shopper lived from the store and the average number of times she/he shopped at the store in a week. (1)

1.2 Calculate the correlation coefficient of the data. (1)

1.3 Calculate the equation of the least squares regression line of the data. (3)

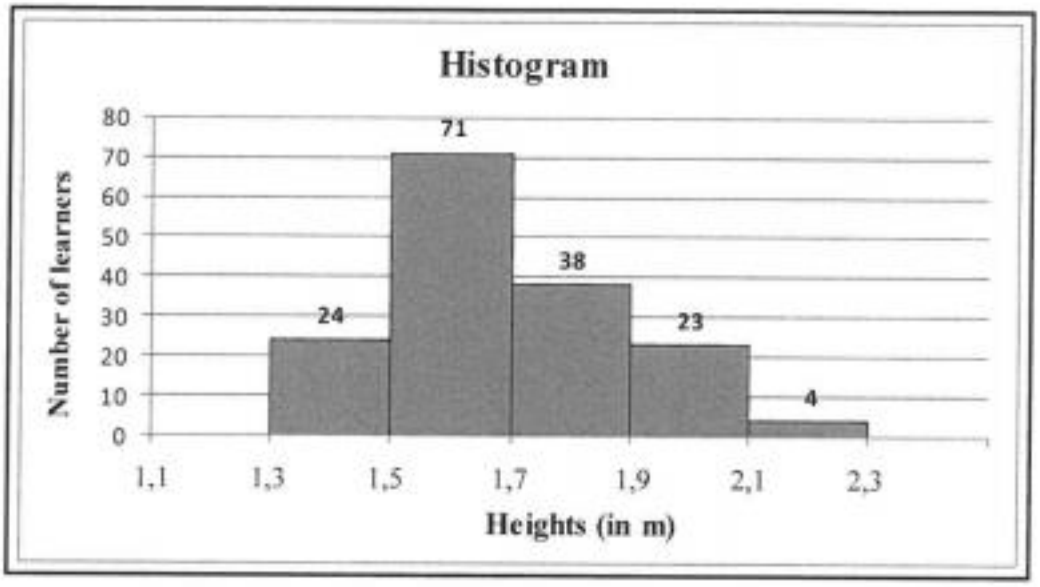
1.4 Use your answer at QUESTION 1.3 to estimate the average number of times that a shopper living 6 km from the supermarket will visit the store in a week. (2)

1.5 Sketch the least squares regression line on the scatter plot provided in the ANSWER BOOK. (2)

[9]

Question 2 **November 2016**

The heights of 160 learners in a school are measured. The height of the shortest learner is 1,39 m and the height of the tallest learner is 2,21 m. The heights are represented in the histogram below.



2.1 Describe the skewness of the data. (1)

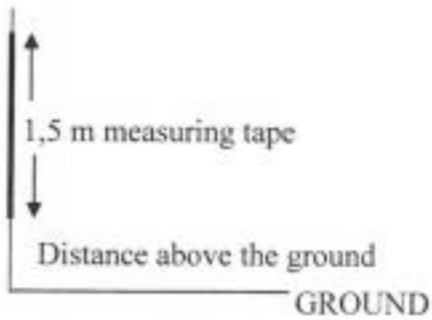
2.2 Calculate the range of the heights. (2)

2.3 Complete the cumulative frequency column in the table given in the ANSWER BOOK. (2)

2.4 Draw an ogive (cumulative frequency curve) to represent the data on the grid provided in the ANSWER BOOK. (4)

2.5 Eighty learners are less than x metres in height. Estimate x. (2)

2.6 The person taking the measurements only had a 1,5 m measuring tape available. In order to compensate for the short measuring tape, he decided to mount the tape on a wall at a height of 1 m above the ground. After recording the measurements he discovered that the tape was mounted at 1,1 m above the ground instead of 1 m.



How does this error influence the following:

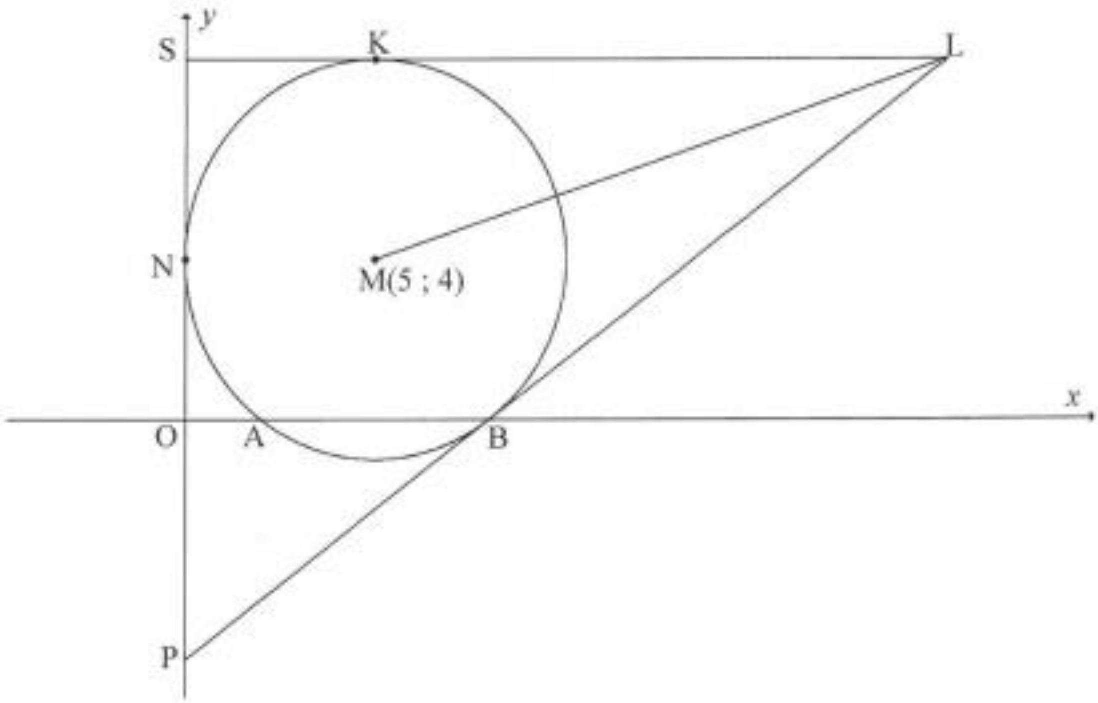
2.6.1 Mean of the data set (1)

2.6.2 Standard deviation of the data set (1)

[13]

Question 3 **November 2014**

In the diagram below, a circle with centre M(5 ; 4) touches the y-axis at N and intersects the x-axis at A and B. PBL and SKL are tangents to the circle where SKL is parallel to the x-axis and P and S are points on the y-axis. LM is drawn.



3.1 Write down the length of the radius of the circle having centre M. (1)

3.2 Write down the equation of the circle having centre M, in the form $(x-a)^2+(y-b)^2=r^2$. (1)

3.3 Calculate the coordinates of A. (3)

3.4 If the coordinates of B are (8 ; 0), calculate:

3.4.1 The gradient of MB (2)

3.4.2 The equation of the tangent PB in the form $y=mx+c$ (3)

3.5 Write down the equation of tangent SKL. (2)

3.6 Show that L is the point (20 ; 9). (2)

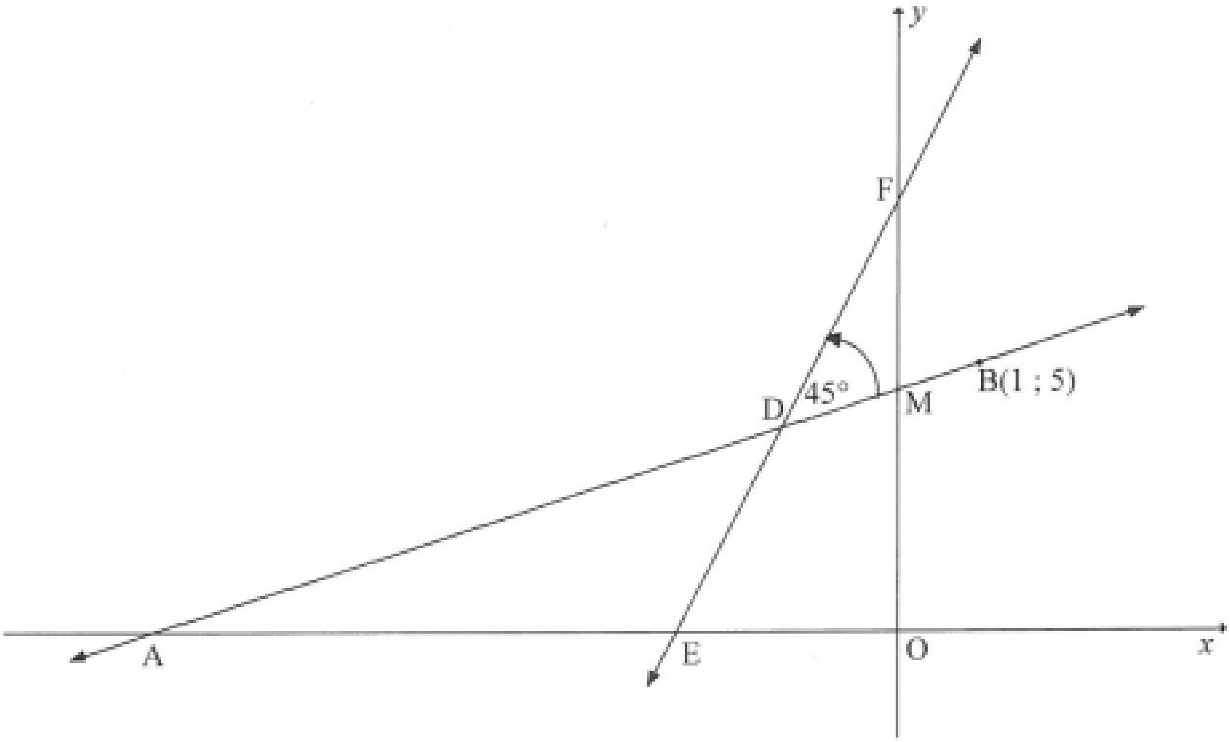
3.7 Calculate the length of ML in surd form. (2)

3.8 Determine the equation of the circle passing through points K, L and M in the form $(x-p)^2+(y-q)^2=c^2$ (5)

[21]

Question 4 **November 2014**

In the diagram below, E and F respectively are the x- and y-intercepts of the line having equation $y = 3x + 8$. The line through B(1 ; 5) making an angle of 45° with EF, as shown below, has x- and y-intercepts A and M respectively.



4.1 Determine the coordinates of E. (2)

4.2 Calculate the size of $D\hat{A}E$. (3)

4.3 Determine the equation of AB in the form $y = mx + c$. (4)

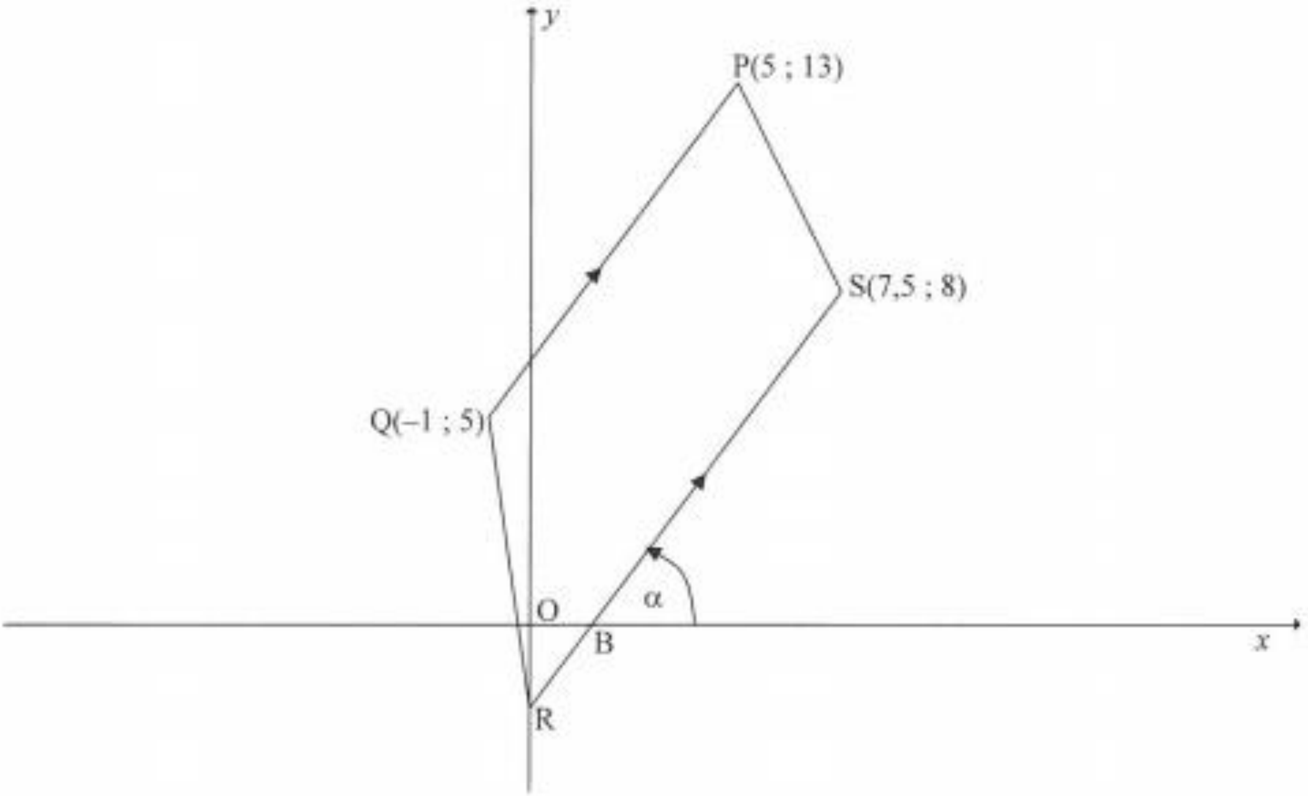
4.4 If AB has equation $x - 2y + 9 = 0$, determine the coordinates of D. (4)

4.5 Calculate the area of quadrilateral DMOE. (6)

[19]

Question 3 **Feb March 2015**

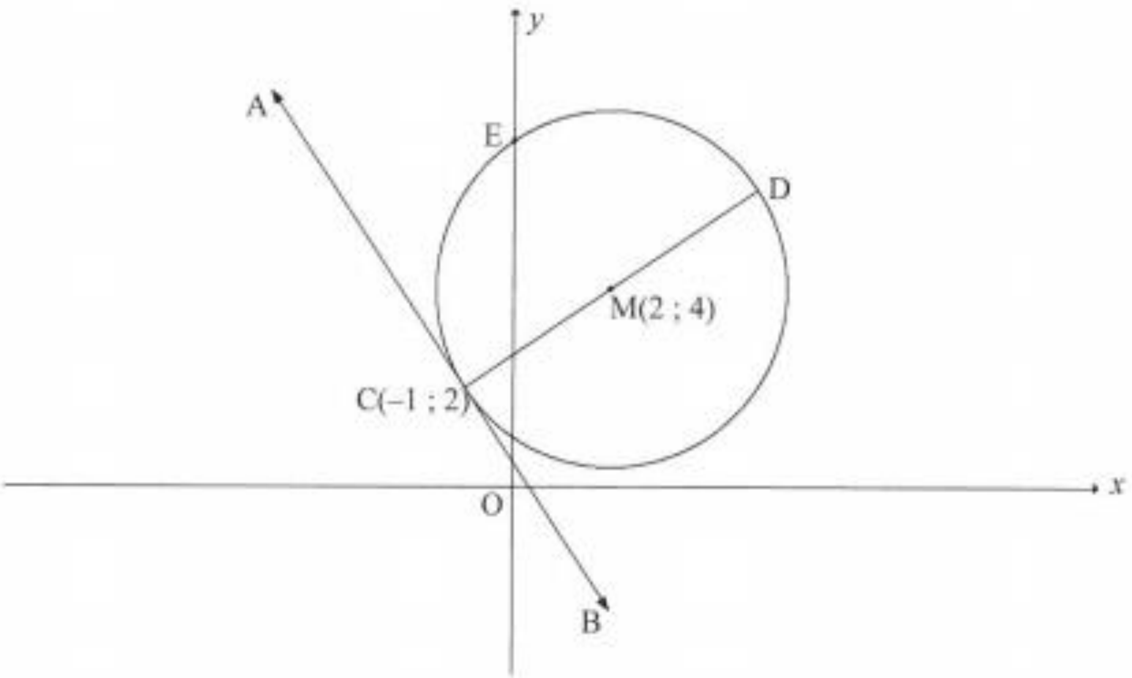
In the diagram below points P(5 ; 13), Q(–1 ; 5) and S(7,5 ; 8) are given. SR | | PQ where R is the y-intercept of SR. The x-intercept of SR is B. QR is joined.



3.1	Calculate the length of PQ.	(3)
3.2	Calculate the gradient of PQ.	(2)
3.3	Determine the equation of line RS in the form $ax + by + c = 0$.	(4)
3.4	Determine the x-coordinate of B.	(2)
3.5	Calculate the size of $O\hat{R}B$.	(3)
3.6	Prove that QBSP is a parallelogram.	(4)
		[18]

Question 4 **Feb March 2015**

4.1 In the diagram below, the circle centred at M(2 ; 4) passes through C(–1 ; 2) and cuts the y-axis at E. The diameter CMD is drawn and ACB is a tangent to the circle.



4.1.1 Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$. (3)

4.1.2 Write down the coordinates of D. (2)

4.1.3 Determine the equation of AB in the form $y=mx+c$. (5)

4.1.4 Calculate the coordinates of E. (4)

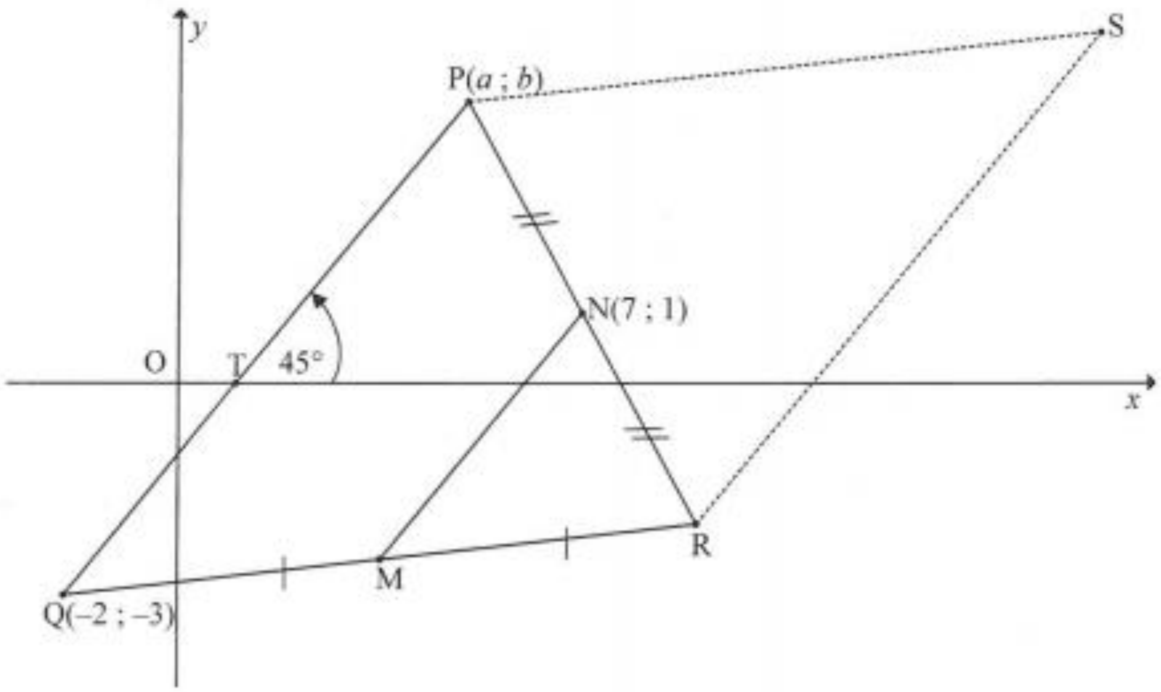
4.1.5 Show that EM is parallel to AB. (2)

4.2 Determine whether or not the circles having equations $(x+2)^2+(y-4)^2=25$ and $(x-5)^2+(y+1)^2=9$ will intersect. Show ALL calculations. (6)

[22]

Question 3 **November 2015**

In the diagram below, the line joining Q(–2 ; –3) and P(a ; b), a and $b > 0$, makes an angle of 45° with the positive x-axis. QP = $7\sqrt{2}$ units. N(7 ; 1) is the midpoint of PR and M is the midpoint of QR.

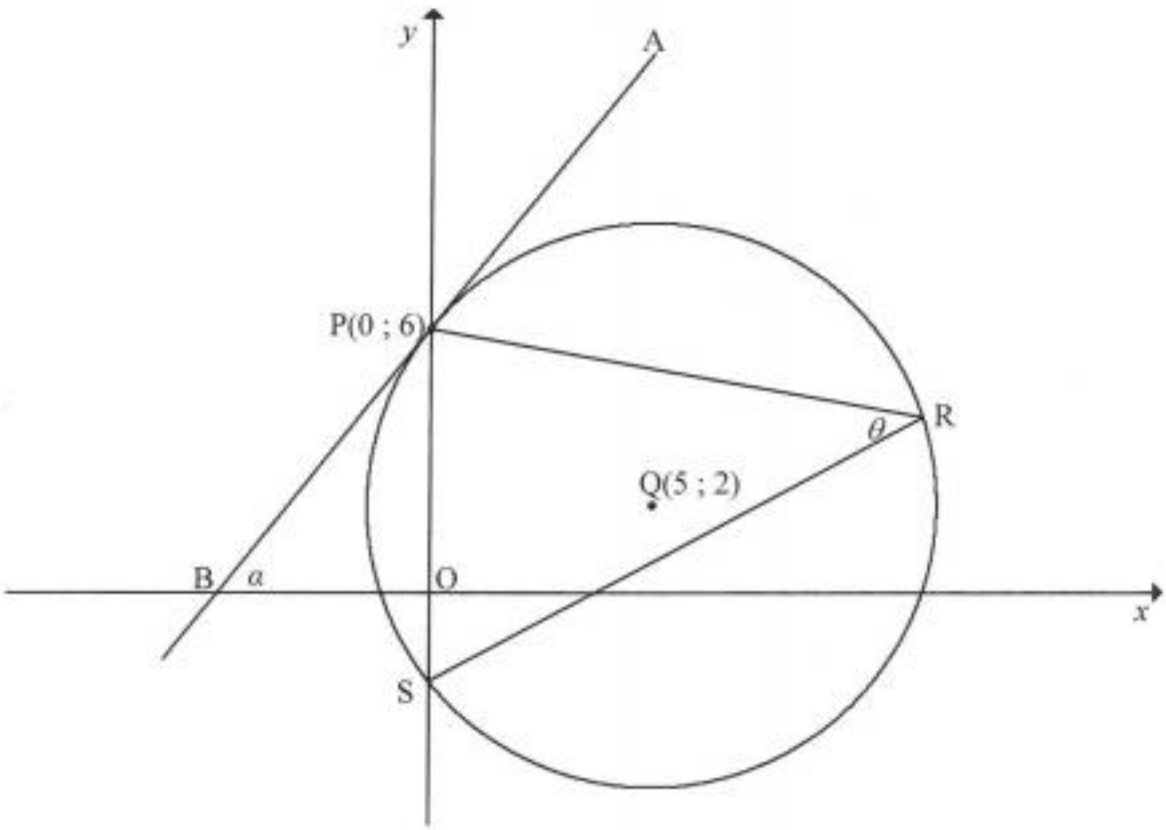


Determine:

3.1	The gradient of PQ	(2)
3.2	The equation of MN in the form $y = mx + c$ and give reasons	(4)
3.3	The length of MN	(2)
3.4	The length of RS	(1)
3.5	The coordinates of S such that PQRS, in this order, is a parallelogram	(3)
3.6	The coordinates of P	(6)
		[18]

Question 4 **November 2015**

In the diagram below, Q(5 ; 2) is the centre of a circle that intersects the y-axis at P(0 ; 6) and S. The tangent APB at P intersects the x-axis at B and makes the angle α with the positive x-axis. R is a point on the circle and $P\hat{R}S = \theta$.



4.1 Determine the equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$. (3)

4.2 Calculate the coordinates of S. (3)

4.3 Determine the equation of the tangent APB in the form $y = mx + c$. (4)

4.4 Calculate the size of α. (2)

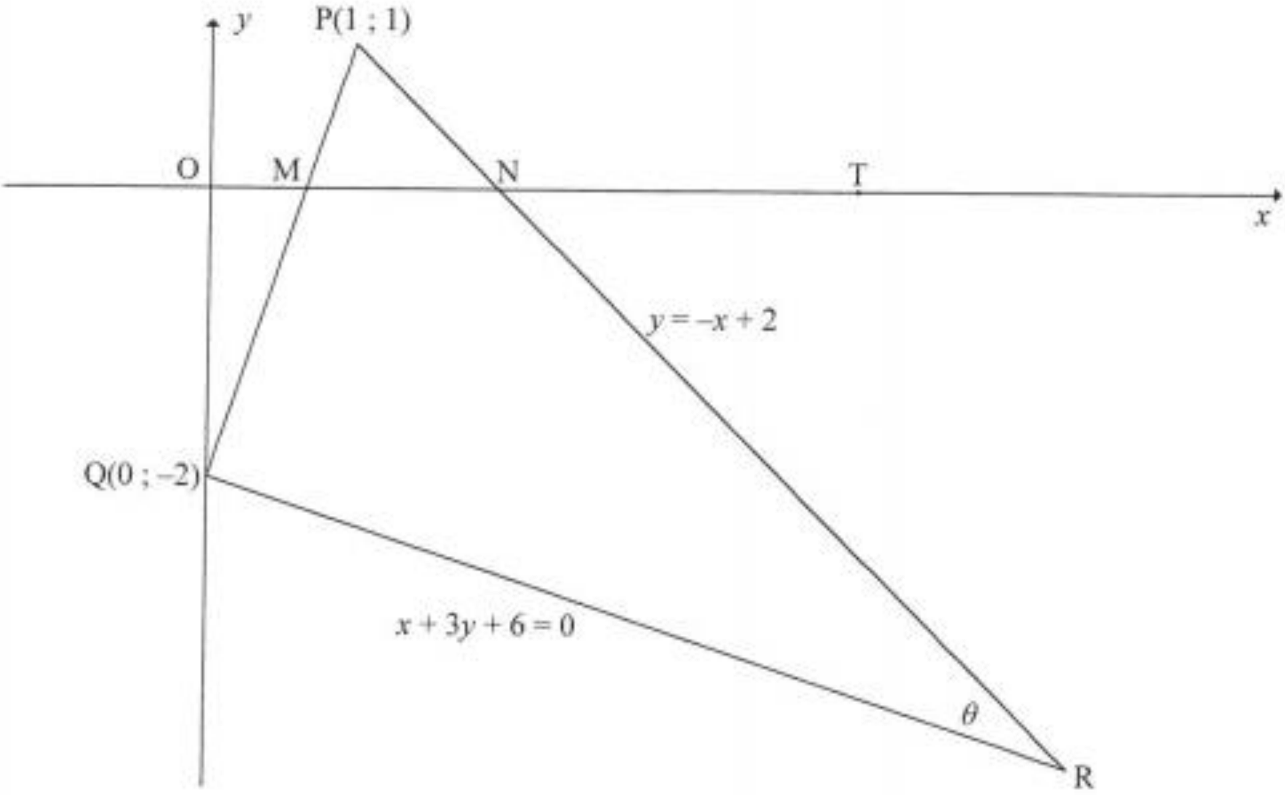
4.5 Calculate, with reasons, the size of θ. (4)

4.6 Calculate the area of ΔPQS. (4)

[20]

Question 3 **Feb March 2016**

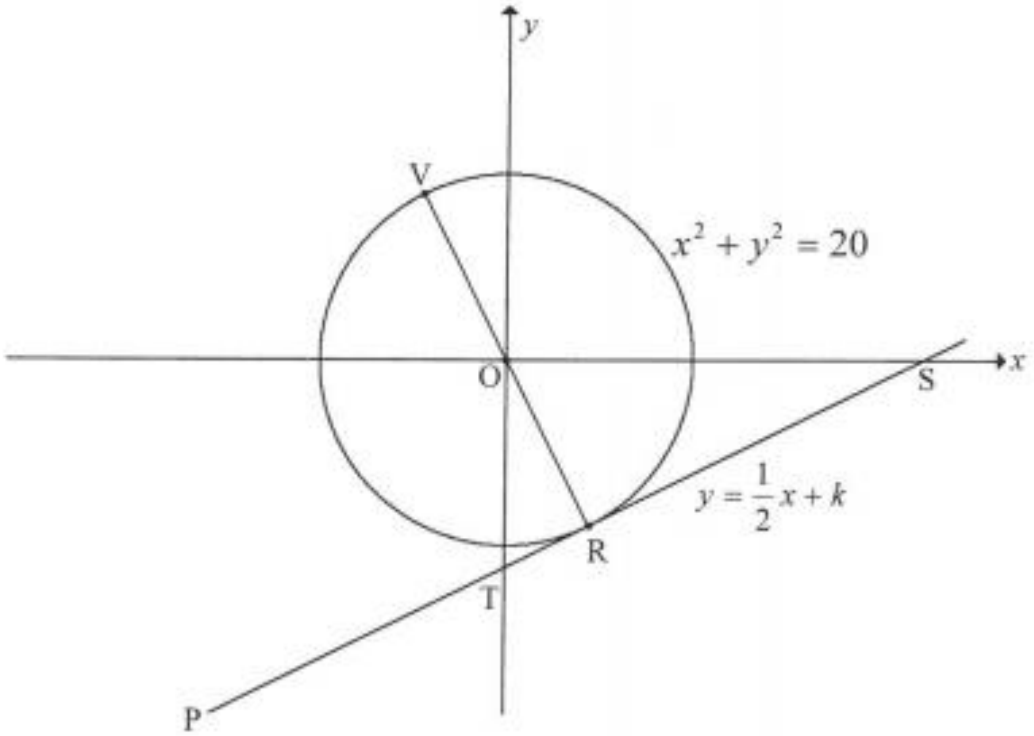
In the diagram below, P(1 ; 1), Q(0 ; –2) and R are the vertices of a triangle and $P\hat{R}Q = \theta$. The x-intercepts of PQ and PR are M and N respectively. The equations of the sides PR and QR are $y = -x + 2$ and $x + 3y + 6 = 0$ respectively. T is a point on the x-axis, as shown.



3.1	Determine the gradient of QP.	(2)
3.2	Prove that $P\hat{Q}R = 90°$.	(2)
3.3	Determine the coordinates of R.	(3)
3.4	Calculate the length of PR. Leave your answer in surd form.	(2)
3.5	Determine the equation of a circle passing through P, Q and R in the form $(x-a)^2 + (y-b)^2 = r^2$.	(6)
3.6	Determine the equation of a tangent to the circle passing through P, Q and R at point P in the form $y = mx + c$.	(3)
3.7	Calculate the size of θ.	(5)
		[23]

Question 4 **Feb March 2016**

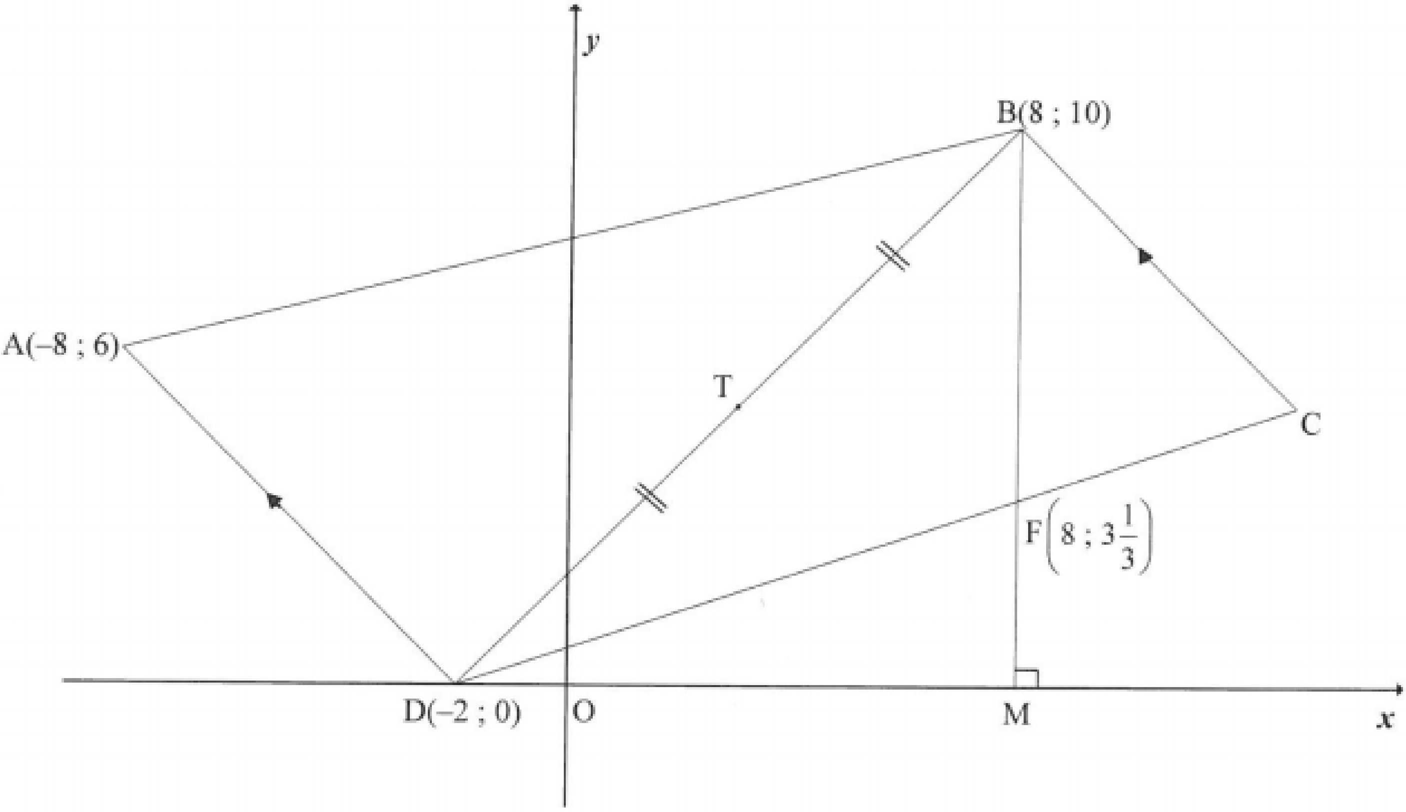
In the diagram below, the equation of the circle with centre O is $x^2 + y^2 = 20$. The tangent PRS to the circle at R has the equation $y = \frac{1}{2}x + k$. PRS cuts the y-axis at T and the x-axis at S.



4.1	Determine, giving reasons, the equation of OR in the form $y = mx + c$.	(3)
4.2	Determine the coordinates of R.	(4)
4.3	Determine the area of ΔOTS, given that R(2 ; –4).	(6)
4.4	Calculate the length of VT.	(4)
		[17]

Question 3 **May June 2016**

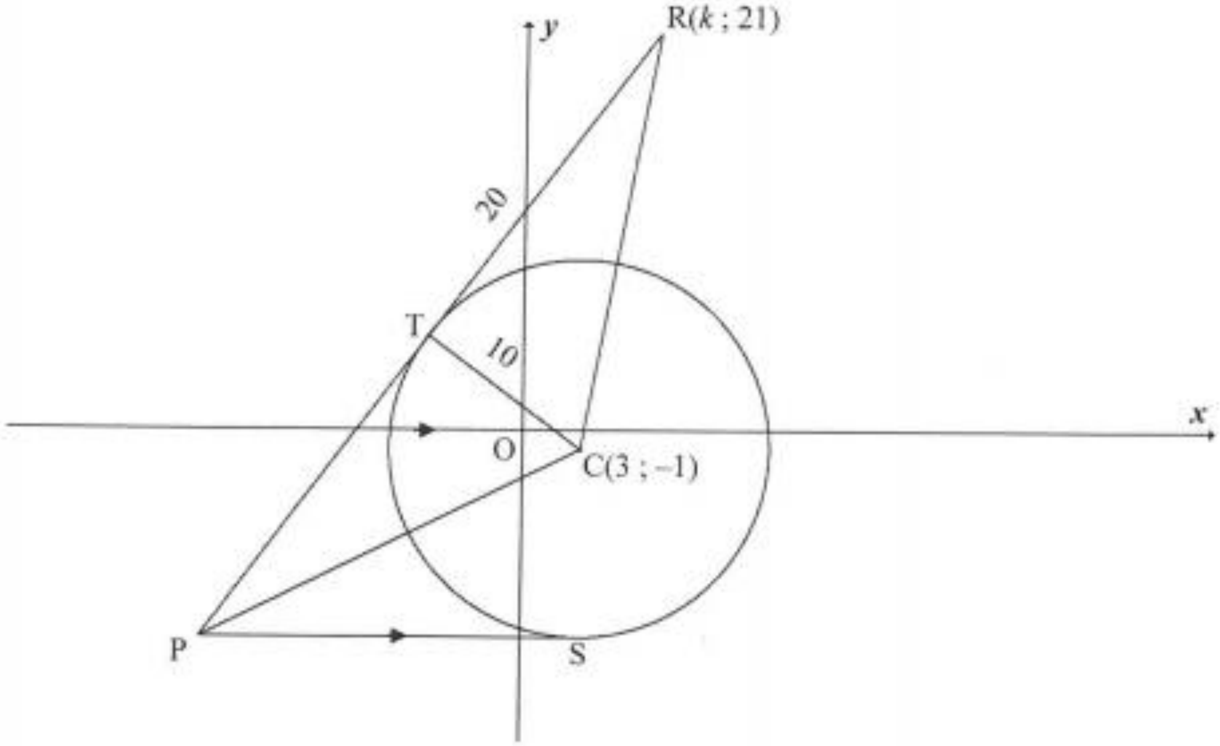
In the diagram below (not drawn to scale) A(–8 ; 6), B(8 ; 10), C and D(–2 ; 0) are the vertices of a trapezium having BC | | AD. T is the midpoint of DB. From B, the straight line drawn parallel to the y-axis cuts DC in $F\left(8 ; 3\frac{1}{3}\right)$ and the x-axis in M.



3.1	Calculate the gradient of AD.	(2)
3.2	Determine the equation of BC in the form $y = mx + c$.	(3)
3.3	Prove that BD ⊥ AD.	(3)
3.4	Calculate the size of $B\hat{D}M$.	(2)
3.5	If it is given that TC | | DM and points T and C are symmetrical about line BM, calculate the coordinates of C.	(3)
3.6	Calculate the area of ΔBDF.	(5)
		[18]

Question 4 **May June 2016**

A circle having C(3 ; −1) as centre and a radius of 10 units is drawn. PTR is a tangent to this circle at T. R(k ; 21), C and P are the vertices of a triangle. TR = 20 units.



4.1 Give a reason why TC ⊥ TR. (1)

4.2 Calculate the length of RC. Leave your answer in surd form. (2)

4.3 Calculate the value of k if R lies in the first quadrant. (4)

4.4 Determine the equation of the circle having centre C and passing through T. Write your answer in the form $(x-a)^2+(y-b)^2=r^2$ (2)

4.5 PS, a tangent to the circle at S, is parallel to the x-axis. Determine the equation of PS. (2)

4.6 The equation of PTR is $3y-4x=35$

4.6.1 Calculate the coordinates of P. (2)

4.6.2 Calculate, giving a reason, the length of PT. (3)

4.7 Consider another circle with equation $(x-3)^2+(y+16)^2=16$ and having centre M.

4.7.1 Write down the coordinates of centre M. (1)

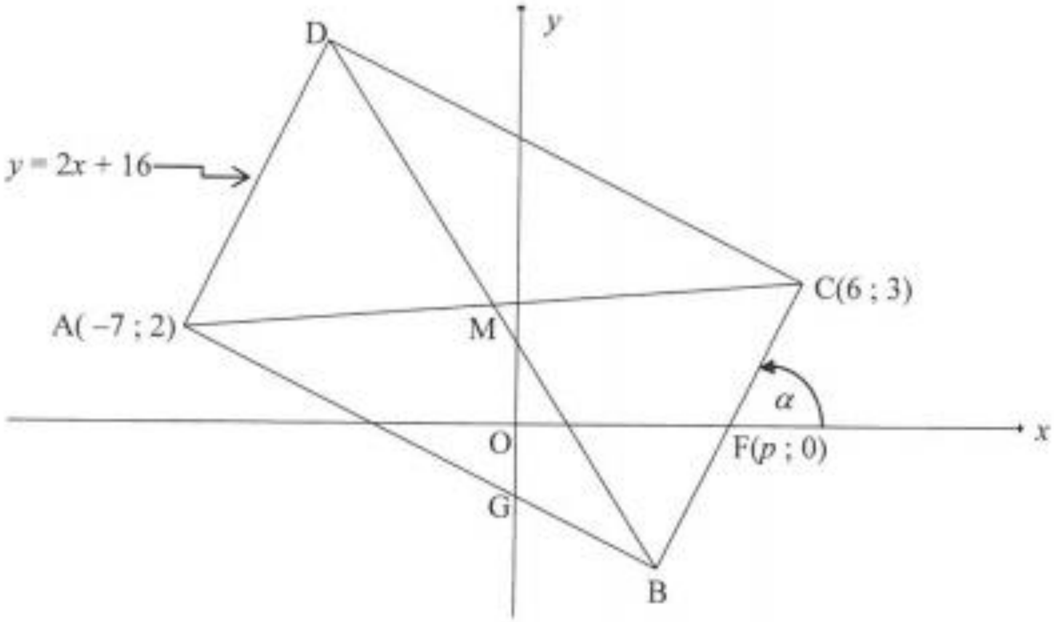
4.7.2 Write down the length of the radius of this circle. (1)

4.7.3 Prove that the circle with centre C and the circle with centre M do not intersect or touch. (3)

[21]

Question 3 **November 2016**

In the diagram, A(–7 ; 2), B, C(6 ; 3) and D are the vertices of rectangle ABCD. The equation of AD is $y = 2x + 16$. Line AB cuts the y-axis at G. The x-intercept of line BC is F(p ; 0) and the angle of inclination of BC with the positive x-axis is α. The diagonals of the rectangle intersect at M.



3.1 Calculate the coordinates of M. (2)

3.2 Write down the gradient of BC in terms of p. (1)

3.3 Hence, calculate the value of p. (3)

3.4 Calculate the length of DB. (3)

3.5 Calculate the size of α (2)

3.6 Calculate the size of $O\hat{G}B$. (3)

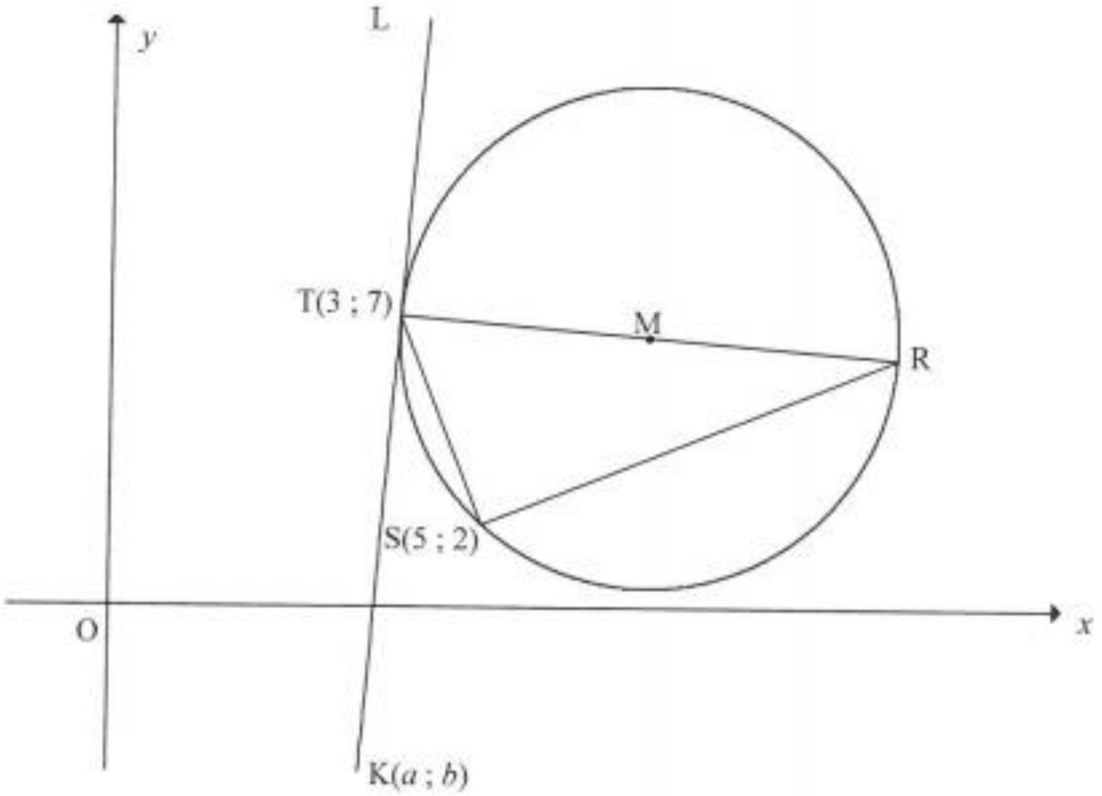
3.7 Determine the equation of the circle passing through points D, B and C in the form $(x-a)^2+(y-b)^2=r^2$. (3)

3.8 If AD is shifted so that ABCD becomes a square, will BC be a tangent to the circle passing through points A, M and B, where M is now the intersection of the diagonals of the square ABCD? Motivate your answer. (2)

[19]

Question 4 **November 2016**

In the diagram, M is the centre of the circle passing through T(3 ; 7), R and S(5 ; 2). RT is a diameter of the circle. K(a ; b) is a point in the 4[th] quadrant such that KTL is a tangent to the circle at T.



4.1 Give a reason why $T\hat{S}R = 90°$. (1)

4.2 Calculate the gradient of TS. (2)

4.3 Determine the equation of the line SR in the form $y = mx + c$. (3)

4.4 The equation of the circle above is $(x-9)^2 + \left(y - 6\frac{1}{2}\right)^2 = 36\frac{1}{4}$.

4.4.1 Calculate the length of TR in surd form. (2)

4.4.2 Calculate the coordinates of R. (3)

4.4.3 Calculate $\sin R$. (3)

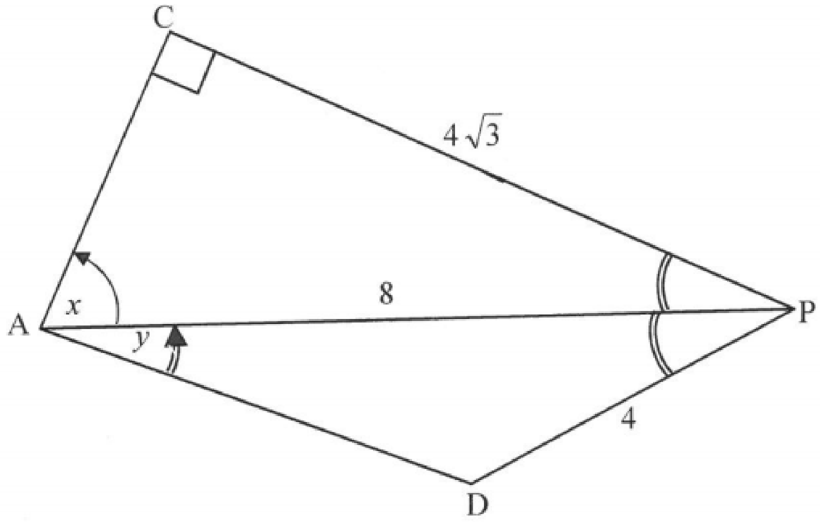
4.4.4 Show that $b = 12a - 29$. (3)

4.4.5 If TK = TR, calculate the coordinates of K. (6)

[23]

Question 5 **November 2014**

In the figure below, ACP and ADP are triangles with $\hat{C} = 90°$, $CP = 4\sqrt{3}$, $AP = 8$ and $DP = 4$. PA bisects $D\hat{P}C$. Let $C\hat{A}P = x$ and $D\hat{A}P = y$.



5.1 Show, by calculation, that $x = 60°$. (2)

5.2 Calculate the length of AD. (4)

5.3 Determine y. (3)

[9]

Question 6 **November 2014**

6.1 Prove the identity: $\cos^2(180° + x) + \tan(x - 180°)\sin(720° - x)\cos x = \cos 2x$ (5)

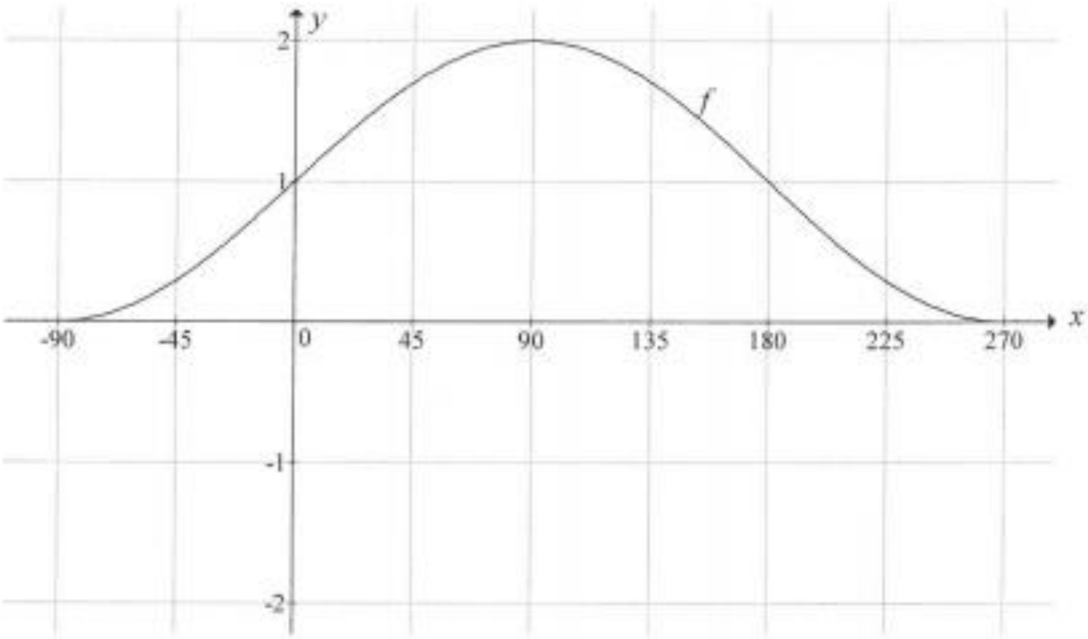
6.2 Use $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ to derive the formula for $\sin(\alpha - \beta)$. (3)

6.3 If $\sin 76° = x$ and $\cos 76° = y$, show that $x^2 - y^2 = \sin 62°$. (4)

[12]

Question 7 **November 2014**

In the diagram below, the graph of $f(x) = \sin x + 1$ is drawn for $-90° \leq x \leq 270°$.



7.1 Write down the range of f. (2)

7.2 Show that $\sin x + 1 = \cos 2x$ can be rewritten as $(2\sin x + 1)\sin x = 0$. (2)

7.3 Hence, or otherwise, determine the general solution of $\sin x + 1 = \cos 2x$. (4)

7.4 Use the grid on DIAGRAM SHEET 2 to draw the graph of $g(x) = \cos 2x$ for $-90° \leq x \leq 270°$. (3)

7.5 Determine the value(s) of x for which $f(x + 30°) = g(x + 30°)$ in the interval $-90° \leq x \leq 270°$. (3)

7.6 Consider the following geometric series:

$1 + 2\cos 2x + 4\cos^2 2x + \ldots$

Use the graph of g to determine the value(s) of x in the interval $0° \leq x \leq 90°$ for which this series will converge. (5)

[19]

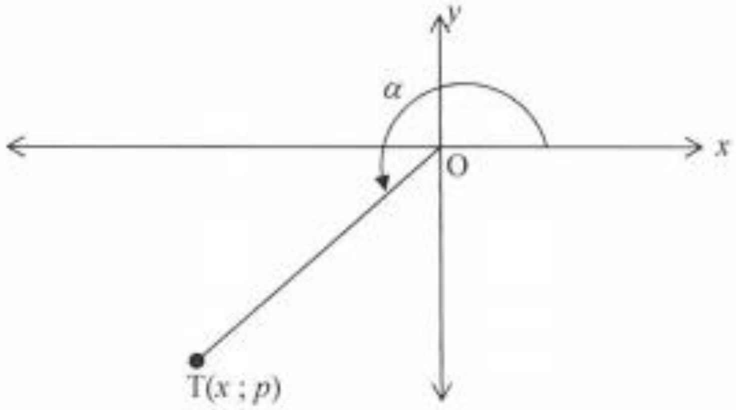
Question 5 — Feb March 2015

5.1 If $x = 3\sin\theta$ and $y = 3\cos\theta$, determine the value of $x^2 + y^2$. (3)

5.2 Simplify to a single term:

$\sin(540° - x).\sin(-x) - \cos(180° - x).\sin(90° + x)$ (6)

5.3 In the diagram below, $T(x\ ;\ p)$ is a point in the third quadrant and it is given that $\sin\alpha = \dfrac{p}{\sqrt{1+p^2}}$.



5.3.1 Show that $x = -1$. (3)

5.3.2 Write $\cos(180° + \alpha)$ in terms of p in its simplest form. (2)

5.3.3 Show that $\cos 2\alpha$ can be written as $\dfrac{1-p^2}{1+p^2}$. (3)

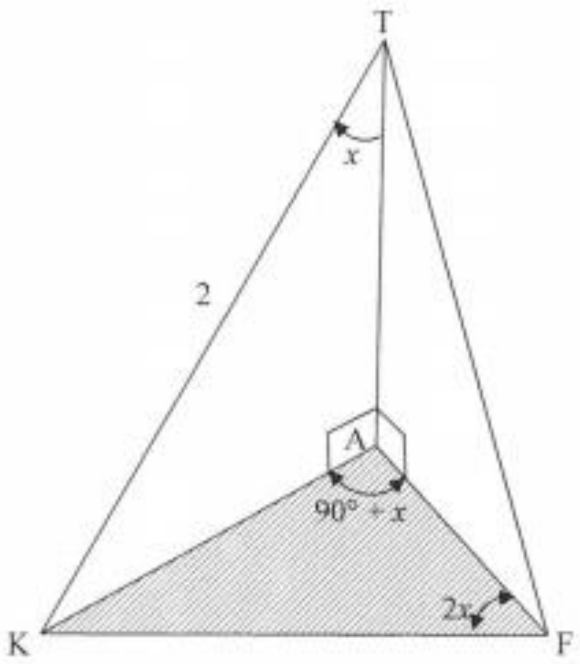
5.4 5.4.1 For which value(s) of x will $\dfrac{2\tan x - \sin 2x}{2\sin^2 x}$ be undefined in the interval $0° \leq x \leq 180°$? (3)

5.4.2 Prove the identity: $\dfrac{2\tan x - \sin 2x}{2\sin^2 x} = \tan x$ (6)

[26]

Question 6 **Feb March 2015**

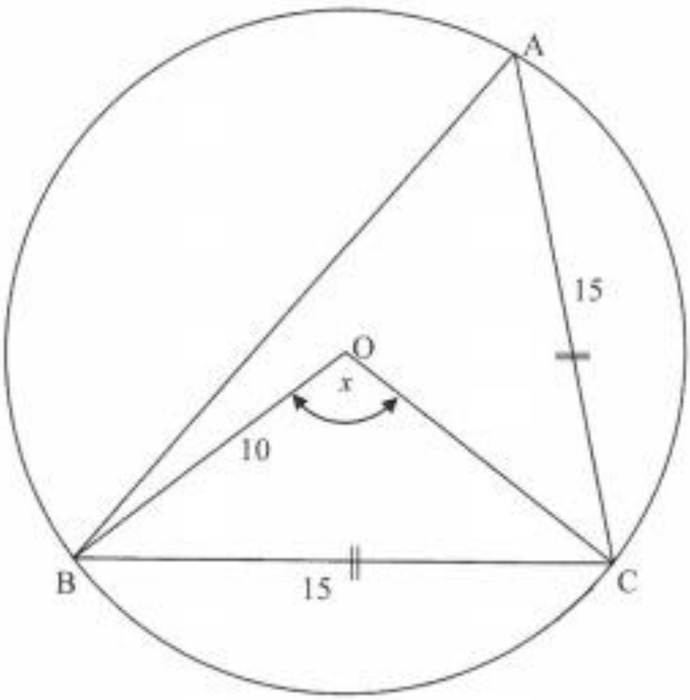
6.1 In the figure, points K, A and F lie in the same horizontal plane and TA represents a vertical tower. $A\hat{T}K = x$, $K\hat{A}F = 90° + x$ and $K\hat{F}A = 2x$ where $0° < x < 30°$. TK = 2 units.



6.1.1 Express AK in terms of $\sin x$. (2)

6.1.2 Calculate the numerical value of KF. (5)

6.2 In the diagram below, a circle with centre O passes through A, B and C. BC = AC = 15 units. BO and OC are joined. OB = 10 units and $B\hat{O}C = x$.



Calculate:

6.2.1 The size of x (4)

6.2.2 The size of $A\hat{C}B$ (3)

6.2.3 The area of ΔABC (2)

[16]

Question 5 **November 2015**

5.1 Given that $\sin 23° = \sqrt{k}$, determine, in its simplest form, the value of each of the following in terms of k, WITHOUT using a calculator:

5.1.1 $\sin 203°$ (2)

5.1.2 $\cos 23°$ (3)

5.1.3 $\tan(-23°)$ (2)

5.2 Simplify the following expression to a single trigonometric function:

$$\frac{4\cos(-x).\cos(90^\circ + x)}{\sin(30^\circ - x).\cos x + \cos(30^\circ - x).\sin x}$$ (6)

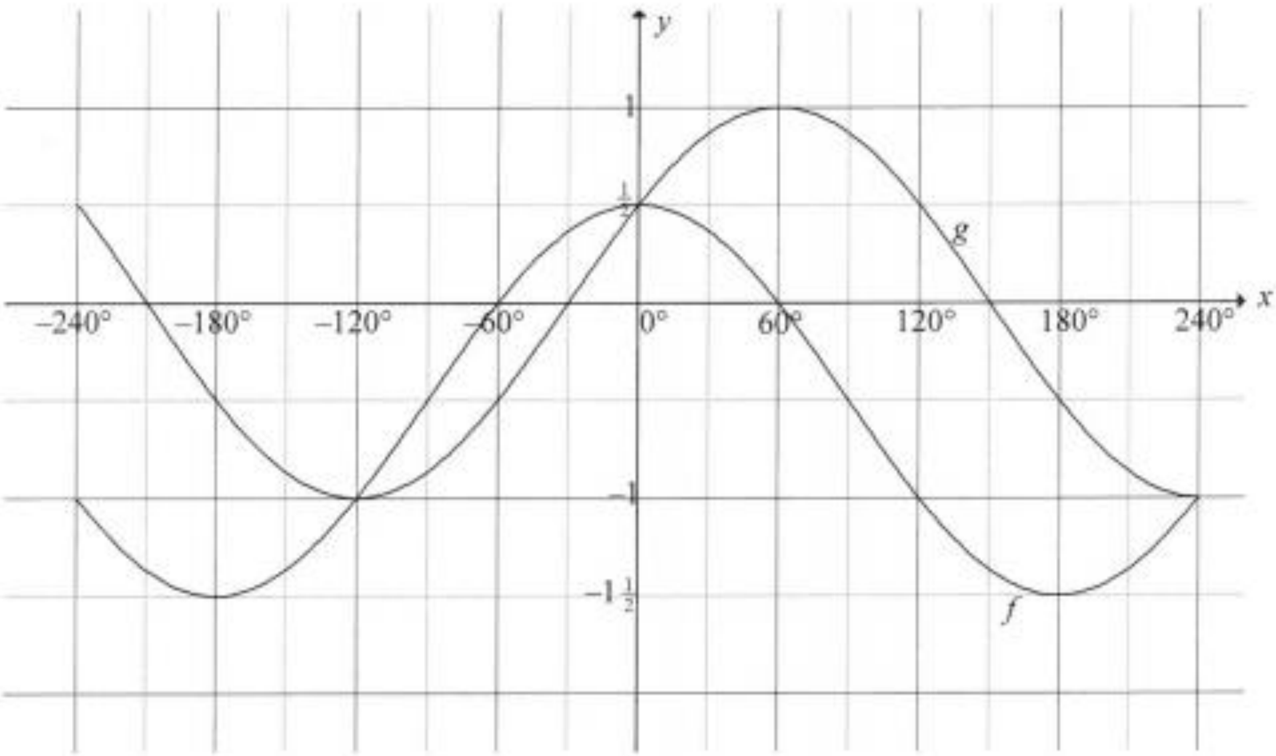
5.3 Determine the general solution of $\cos 2x - 7\cos x - 3 = 0$. (6)

5.4 Given that $\sin\theta = \frac{1}{3}$, calculate the numerical value of $\sin 3\theta$, WITHOUT using a calculator. (5)

[24]

Question 6 **November 2015**

In the diagram below, the graphs of $f(x) = \cos x + q$ and $g(x) = \sin(x + p)$ are drawn on the same system of axes for $-240^\circ \leq x \leq 240^\circ$. The graphs intersect at $\left(0^\circ ; \frac{1}{2}\right)$, $(-120^\circ ; -1)$ and $(240^\circ ; -1)$.



6.1 Determine the values of p and q. (4)

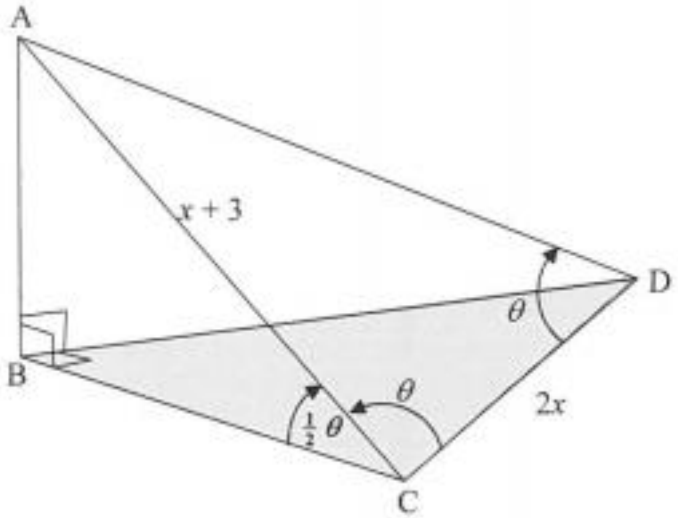
6.2 Determine the values of x in the interval $-240^\circ \leq x \leq 240^\circ$ for which $f(x) > g(x)$. (2)

6.3 Describe a transformation that the graph of g has to undergo to form the graph of h, where $h(x) = -\cos x$. (2)

[8]

Question 7 **November 2015**

A corner of a rectangular block of wood is cut off and shown in the diagram below. The inclined plane, that is, ΔACD, is an isosceles triangle having $A\hat{D}C = A\hat{C}D = \theta$. Also $A\hat{C}B = \frac{1}{2}\theta$, $AC = x + 3$ and $CD = 2x$.



7.1 Determine an expression for $C\hat{A}D$ in terms of θ. (1)

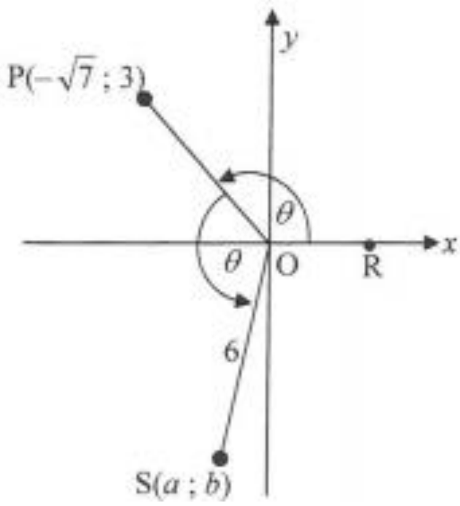
7.2 Prove that $\cos\theta = \dfrac{x}{x+3}$. (4)

7.3 If it is given that $x = 2$, calculate AB, the height of the piece of wood. (5)

[10]

Question 5 **Feb March 2016**

5.1 $P(-\sqrt{7}\,;3)$ and $S(a\,;b)$ are points on the Cartesian plane, as shown in the diagram below. $P\hat{O}R = P\hat{O}S = \theta$ and $OS = 6$.



Determine, WITHOUT using a calculator, the value of:

5.1.1 $\tan\theta$ (1)

5.1.2 $\sin(-\theta)$ (3)

5.1.3 a (4)

5.2 5.2.1 Simplify $\dfrac{4\sin x\cos x}{2\sin^2 x-1}$ to a single trigonometric ratio. (3)

5.2.2 Hence, calculate the value of $\dfrac{4\sin15^\circ\cos15^\circ}{2\sin^2 15^\circ-1}$ WITHOUT using a calculator. (Leave your answer in simplest surd form.) (2)

[13]

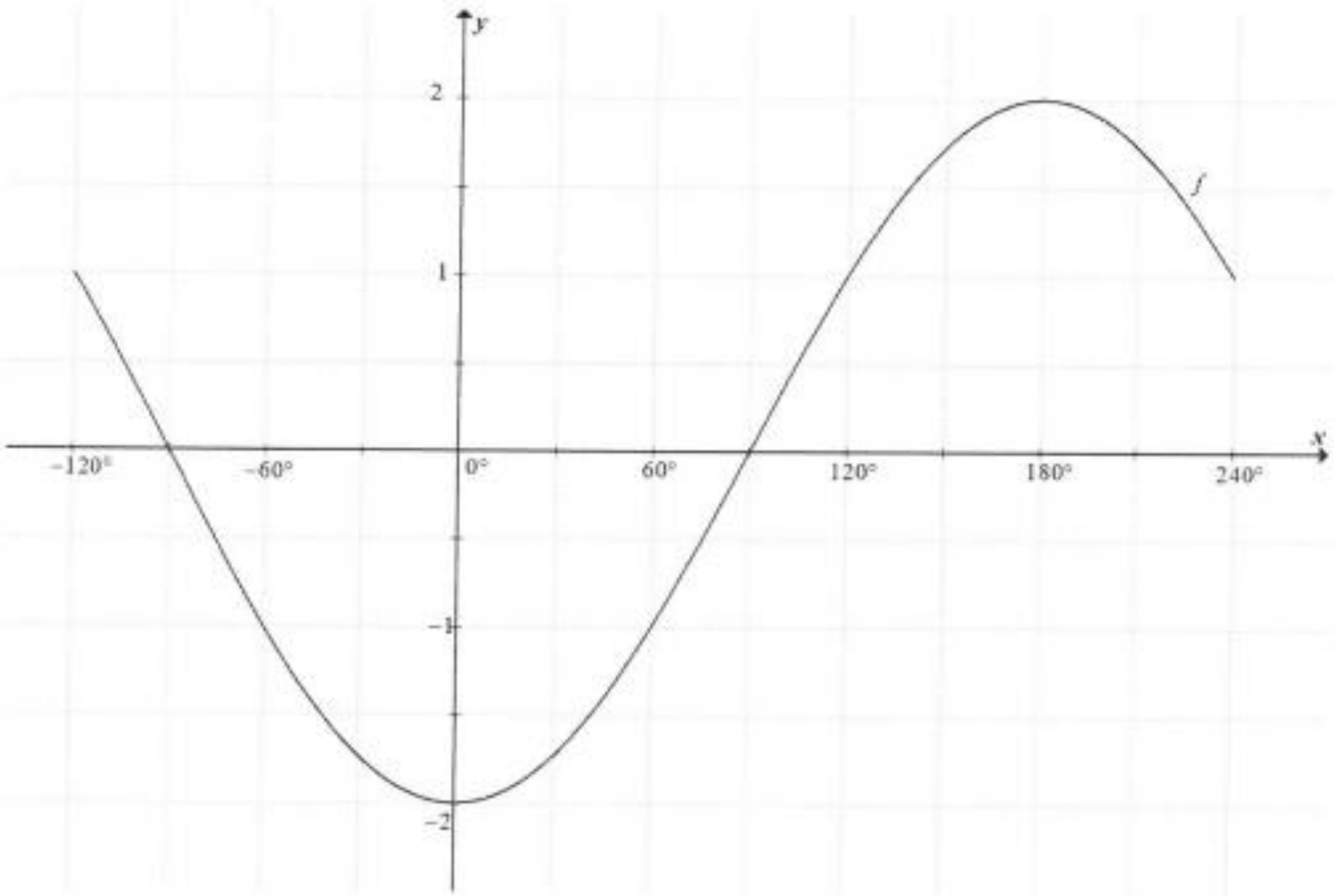
Question 6 **Feb March 2016**

Given the equation: $\sin(x+60^\circ)+2\cos x=0$

6.1 Show that the equation can be rewritten as $\tan x=-4-\sqrt{3}$. (4)

6.2 Determine the solutions of the equation $\sin(x+60^\circ)+2\cos x=0$ in the interval $-180^\circ\leq x\leq 180^\circ$. (3)

6.3 In the diagram below, the graph of $f(x)=-2\cos x$ is drawn for $-120^\circ\leq x\leq 240^\circ$.



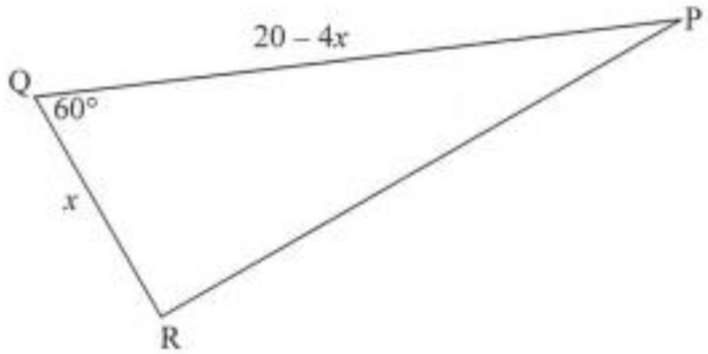
6.3.1 Draw the graph of $g(x) = \sin(x + 60°)$ for $-120° \leq x \leq 240°$ on the grid provided in the ANSWER BOOK. (3)

6.3.2 Determine the values of x in the interval $-120° \leq x \leq 240°$ for which $\sin(x + 60°) + 2\cos x > 0$. (3)

[13]

Question 7 **Feb March 2016**

7.1 In the diagram below, ΔPQR is drawn with $PQ = 20 - 4x$, $RQ = x$ and $\hat{Q} = 60°$.

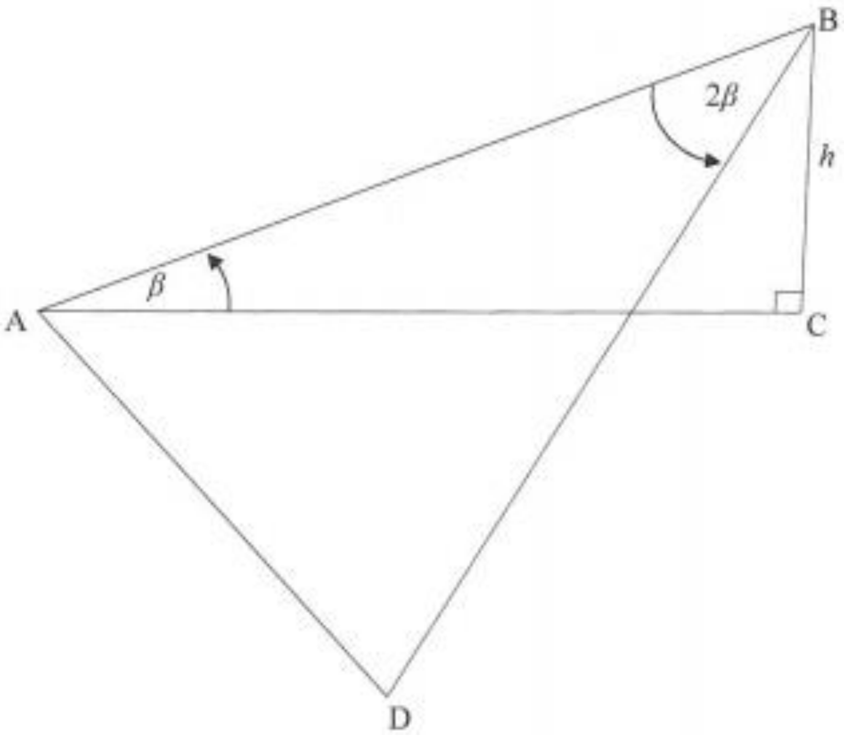


7.1.1 Show that the area of $\Delta PQR = 5\sqrt{3}x - \sqrt{3}x^2$. (2)

7.1.2 Determine the value of x for which the area of ΔPQR will be a maximum. (3)

7.1.3 Calculate the length of PR if the area of ΔPQR is a maximum. (3)

7.2 In the diagram below, BC is a pole anchored by two cables at A and D. A, D and C are in the same horizontal plane. The height of the pole is h and the angle of elevation from A to the top of the pole, B, is β. $A\hat{B}D = 2\beta$ and $BA = BD$.

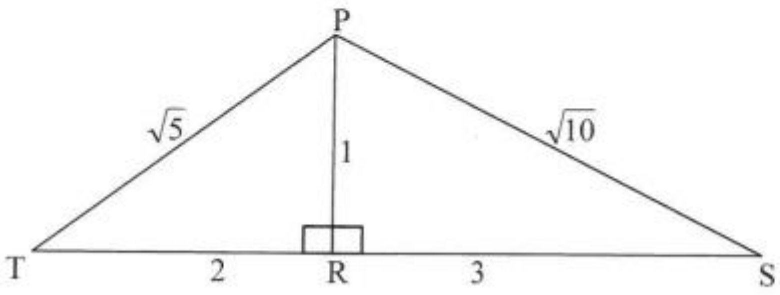


Determine the distance AD between the two anchors in terms of h. (7)

[15]

Question 5 **May June 2016**

5.1 In the diagram $PR \perp TS$ in obtuse triangle PTS.
$PT = \sqrt{5}$; $TR = 2$; $PR = 1$; $PS = \sqrt{10}$ and $RS = 3$



5.1.1 Write down the value of:

(a) $\sin \hat{T}$ (1)

(b) $\cos \hat{S}$ (1)

5.1.2 Calculate, WITHOUT using a calculator, the value of $\cos(\hat{T} + \hat{S})$ (5)

5.2 Determine the value of:

$$\frac{1}{\cos(360° - \theta).\sin(90° - \theta)} - \tan^2(180° + \theta)$$ (6)

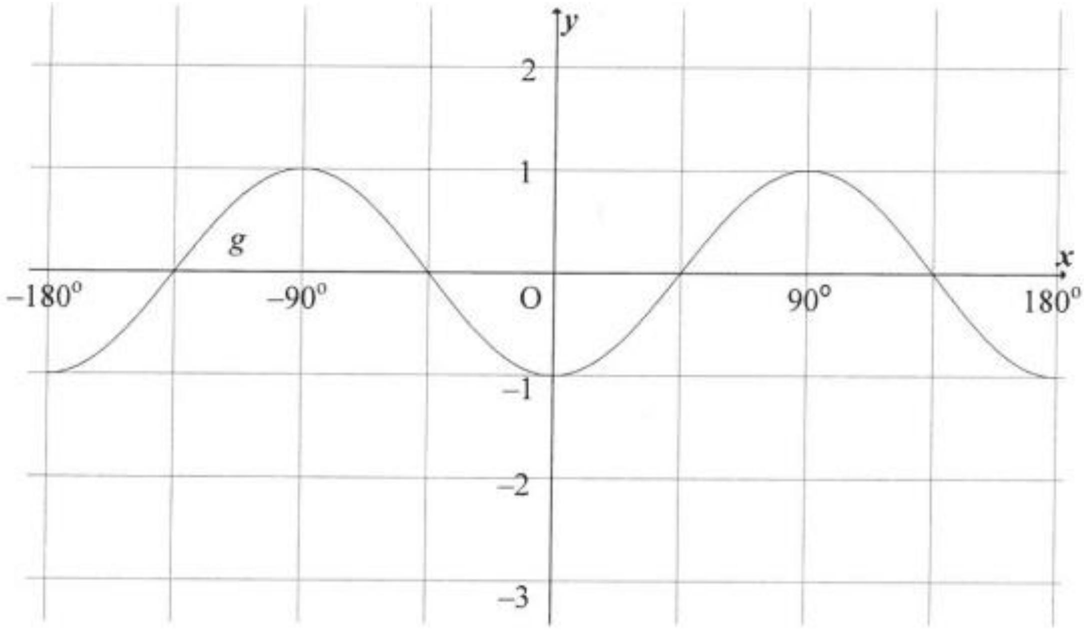
5.3 If $\sin x - \cos x = \frac{3}{4}$, calculate the value of $\sin 2x$ WITHOUT using a calculator. (5)

[18]

Question 6 **May June 2016**

6.1 Determine the general solution of $4\sin x + 2\cos 2x = 2$ (6)

6.2 The graph of $g(x) = -\cos 2x$ for $x \in [-180° ; 180°]$ is drawn below.



6.2.1 Draw the graph of $f(x) = 2\sin x - 1$ for $x \in [-180° ; 180°]$ on the set of axes provided in the ANSWER BOOK. (3)

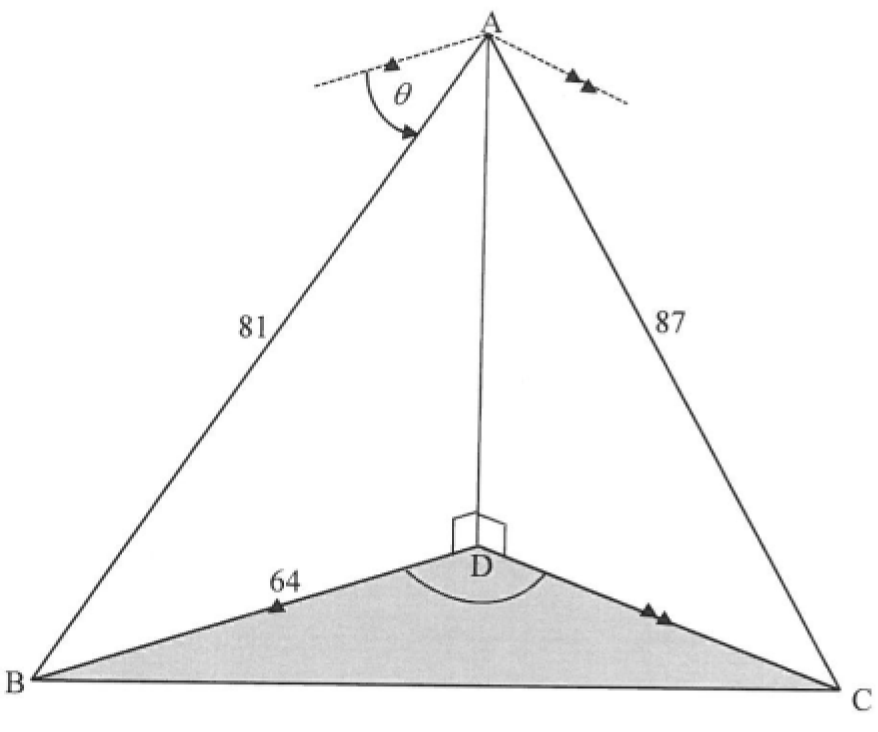
6.2.2 Write down the values of x for which g is strictly decreasing in the interval $x \in [-180° ; 0°]$ (2)

6.2.3 Write down the value(s) of x for which $f(x + 30°) - g(x + 30°) = 0$ for $x \in [-180° ; 180°]$ (2)

[13]

Question 7 **May June 2016**

From point A an observer spots two boats, B and C, at anchor. The angle of depression of boat B from A is θ. D is a point directly below A and is on the same horizontal plane as B and C. BD = 64 m, AB = 81 m and AC = 87 m.



7.1 Calculate the size of θ to the nearest degree. (3)

7.2 If it is given that $B\hat{A}C = 82{,}6°$, calculate BC, the distance between the boats. (3)

7.3 If $B\hat{D}C = 110°$, calculate the size of $D\hat{C}B$. (3)

[9]

Question 5 **November 2016**

5.1 Given: $\sin 16° = p$

Determine the following in terms of p, **without using a calculator**.

5.1.1 $\sin 196°$ (2)

5.1.2 $\cos 16°$ (2)

5.2 Given: $\cos(A - B) = \cos A \cos B + \sin A \sin B$

Use the formula for $\cos(A - B)$ to derive a formula for $\sin(A + B)$ (3)

5.3 Simplify $\dfrac{\sqrt{1 - \cos^2 2A}}{\cos(-A).\cos(90° + A)}$ completely, given that $0° < A < 90°$. (5)

5.4 Given: $\cos 2B = \dfrac{3}{5}$ and $0° \leq B \leq 90°$

Determine, **without using a calculator**, the value of EACH of the following in its simplest form:

5.4.1 $\cos B$ (3)

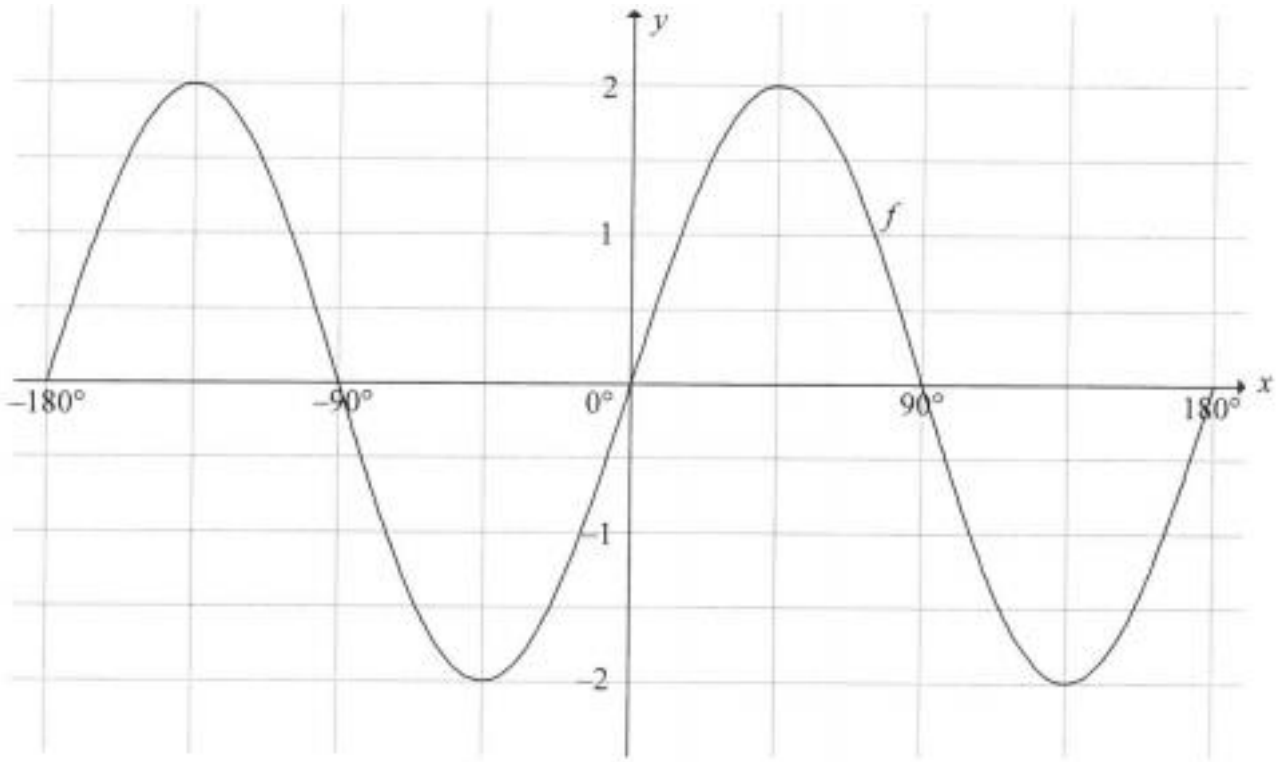
5.4.2 $\sin B$ (2)

5.4.3 $\cos(B + 45°)$ (4)

[21]

Question 6 **November 2016**

In the diagram the graph of $f(x) = 2\sin 2x$ is drawn for the interval $x \in [-180°; 180°]$.



6.1 On the system of axes on which f is drawn in the ANSWER BOOK, draw the graph of $g(x) = -\cos 2x$ for $x \in [-180°\,;\,180°]$. Clearly show all intercepts with the axes, the coordinates of the turning points and end points of the graph. (3)

6.2 Write down the maximum value of $f(x) - 3$. (2)

6.3 Determine the general solution of $f(x) = g(x)$. (4)

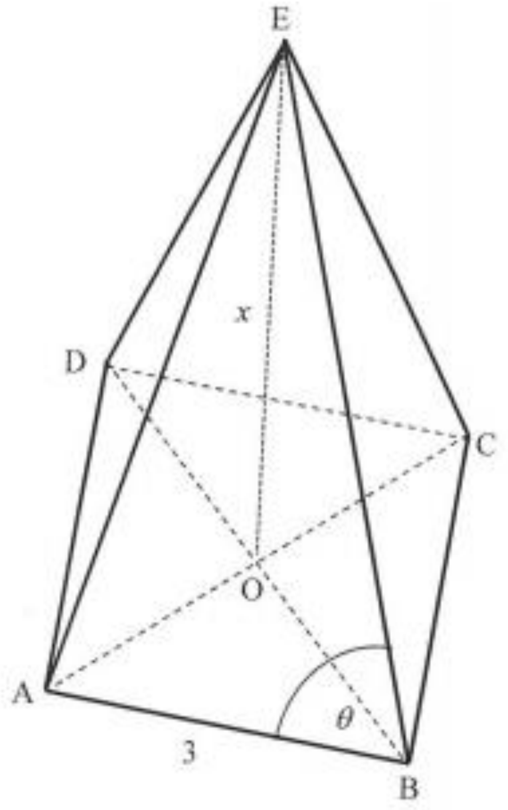
6.4 Hence, determine the values of x for which $f(x) < g(x)$ in the interval $x \in [-180°\,;\,0°]$. (3)

[12]

Question 7 **November 2016**

E is the apex of a pyramid having a square base ABCD. O is the centre of the base. $E\hat{B}A = \theta$, AB = 3 m and EO, the perpendicular height of the pyramid, is x.

Volume of pyramid $= \frac{1}{3}$(area of base) × ($\perp$ height)



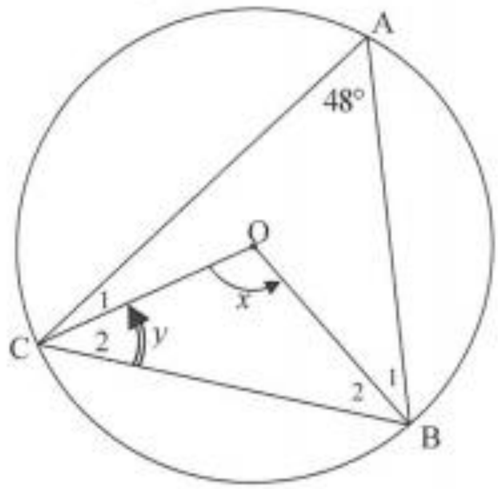
7.1 Calculate the length of OB. (3)

7.2 Show that $\cos\theta = \dfrac{3}{2\sqrt{x^2 + \dfrac{9}{2}}}$ (5)

7.3 If the volume of the pyramid is $15\ \text{m}^3$, calculate the value of θ. (4)
[12]

Question 8 **November 2014**

8.1 In the diagram, O is the centre of the circle passing through A, B and C. $C\hat{A}B = 48°$, $C\hat{O}B = x$ and $\hat{C}_2 = y$.

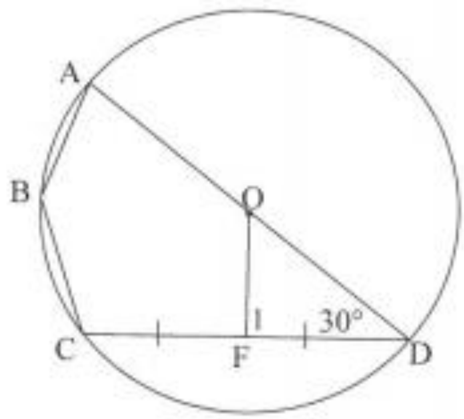


Determine, with reasons, the size of:

8.1.1 x (2)

8.1.2 y (2)

8.2 In the diagram, O is the centre of the circle passing through A, B, C and D. AOD is a straight line and F is the midpoint of chord CD. $O\hat{D}F = 30°$ and OF are joined.

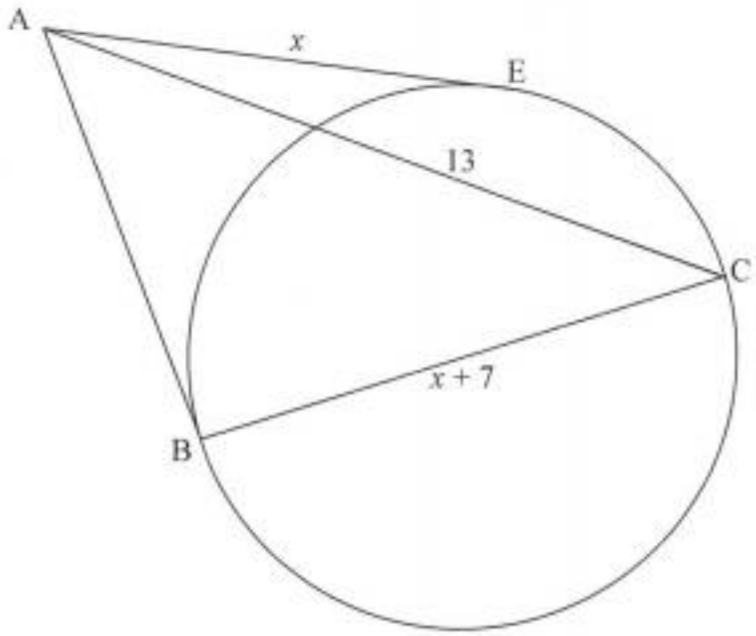


Determine, with reasons, the size of:

8.2.1 $\hat{F}_1$ (2)

8.2.2 $A\hat{B}C$ (2)

8.3 In the diagram, AB and AE are tangents to the circle at B and E respectively. BC is a diameter of the circle. AC = 13, AE = x and BC = $x + 7$.



8.3.1 Give reasons for the statements below.
Complete the table on DIAGRAM SHEET 3.

	Statement	**Reason**
(a)	$A\hat{B}C = 90°$	
(b)	AB = x	

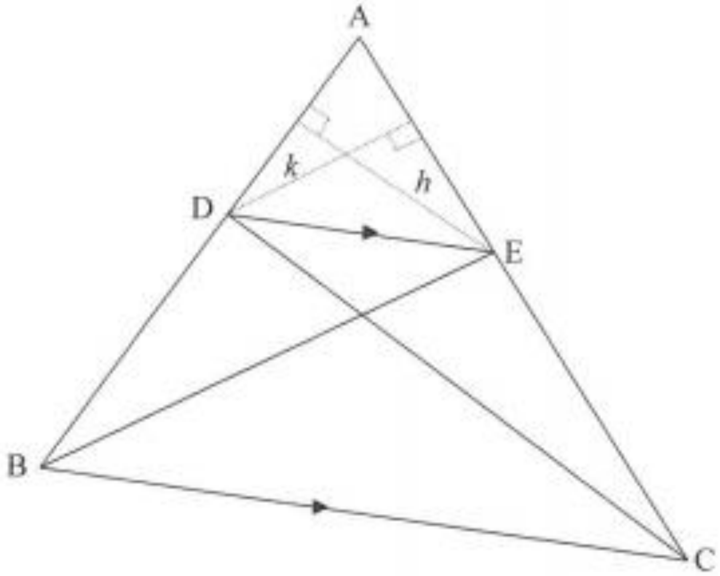
(2)

8.3.2 Calculate the length of AB. (4)

[14]

Question 9 **November 2014**

9.1 In the diagram, points D and E lie on sides AB and AC of ΔABC respectively such that $DE \parallel BC$. DC and BE are joined.



9.1.1 Explain why the areas of ΔDEB and ΔDEC are equal. (1)

9.1.2 Given below is the partially completed proof of the theorem that states that if in any ΔABC the line $DE \parallel BC$ then $\frac{AD}{DB} = \frac{AE}{EC}$.

Using the above diagram, complete the proof of the theorem on DIAGRAM SHEET 4.

Construction: Construct the altitudes (heights) h and k in ΔADE.

$$\frac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \frac{\frac{1}{2}(AD)(h)}{\frac{1}{2}(BD)(h)} = \text{.........}$$

$$\frac{\text{area } \Delta ADE}{\text{area } \Delta DEC} = \text{........................} = \frac{AE}{EC}$$

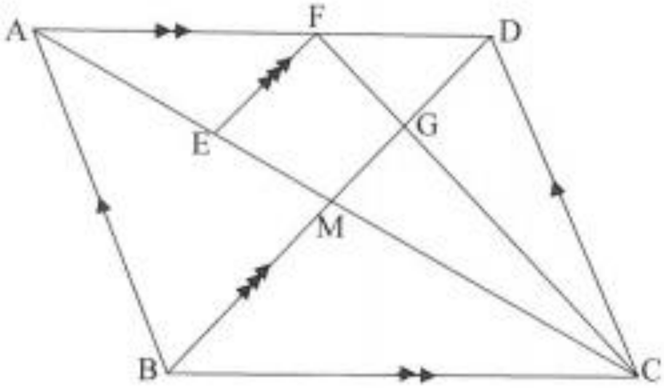
But area ΔDEB = ... (reason: ....)

$$\therefore \frac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \text{................................}$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

(5)

9.2 In the diagram, ABCD is a parallelogram. The diagonals of ABCD intersect in M. F is a point on AD such that AF : FD = 4 : 3. E is a point on AM such that EF || BD. FC and MD intersect in G.



Calculate, giving reasons, the ratio of:

9.2.1 $\frac{EM}{AM}$ (3)

9.2.2 $\frac{CM}{ME}$ (3)

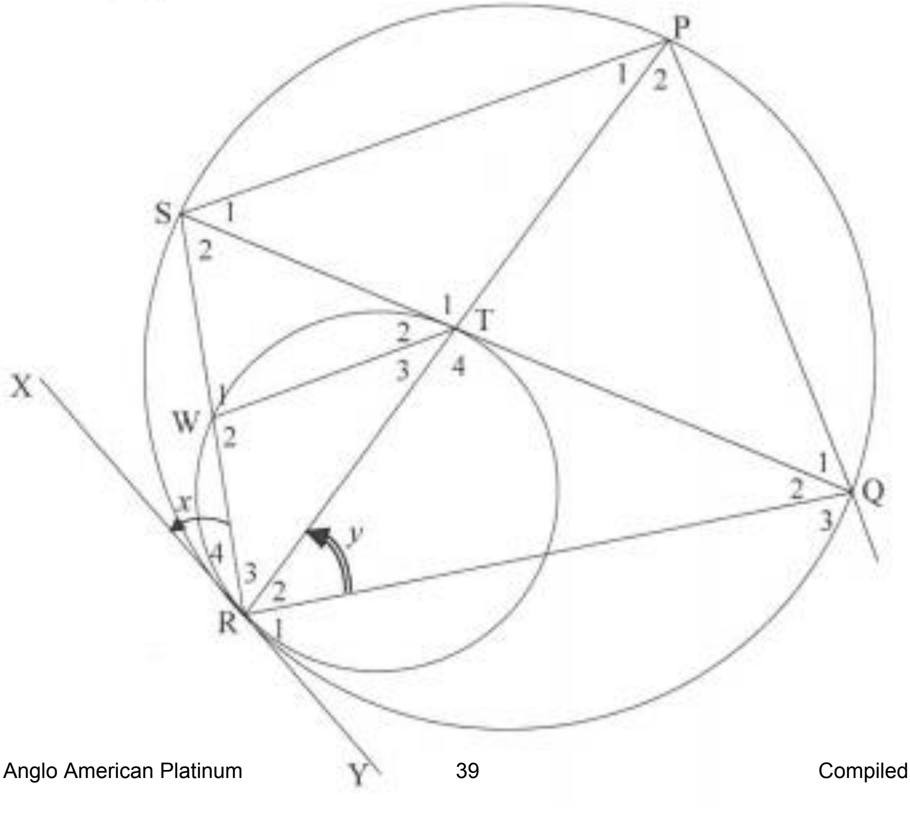
9.2.3 $\frac{\text{area } \Delta FDC}{\text{area } \Delta BDC}$ (4)

[16]

Question 10 **November 2014**

The two circles in the diagram have a common tangent XRY at R. W is any point on the small circle. The straight line RWS meets the large circle at S. The chord STQ is a tangent to the small circle, where T is the point of contact. Chord RTP is drawn.

Let $\hat{R}_4 = x$ and $\hat{R}_2 = y$



10.1 Give reasons for the statements below.
Complete the table on DIAGRAM SHEET 6.

Let $\hat{R}_4 = x$ and $\hat{R}_2 = y$		
	Statement	**Reason**
10.1.1	$\hat{T}_3 = x$	
10.1.2	$\hat{P}_1 = x$	
10.1.3	WT || SP	
10.1.4	$\hat{S}_1 = y$	
10.1.5	$\hat{T}_2 = y$	

(5)

10.2 Prove that $RT = \frac{WR.RP}{RS}$ (2)

10.3 Identify, with reasons, another TWO angles equal to y. (4)

10.4 Prove that $\hat{Q}_3 = \hat{W}_2$. (3)

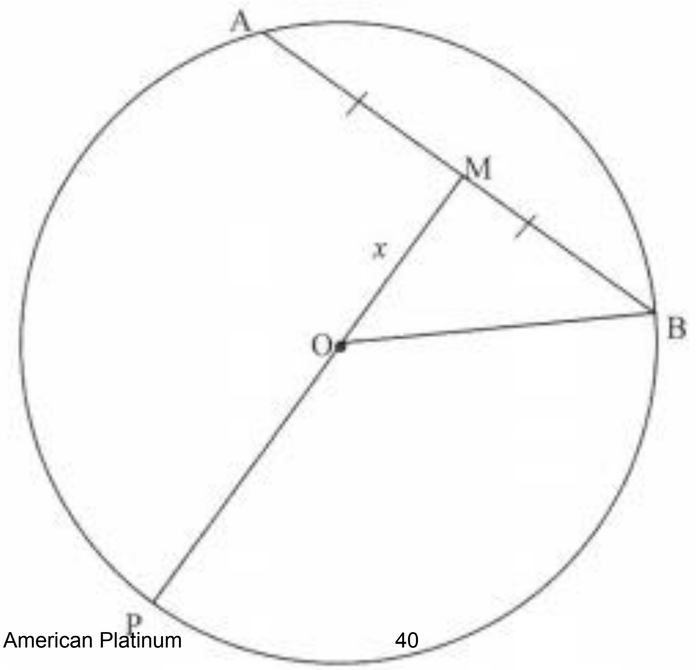
10.5 Prove that $\Delta RTS \,|||\, \Delta RQP$. (3)

10.6 Hence, prove that $\frac{WR}{RQ} = \frac{RS^2}{RP^2}$. (3)

[20]

Question 7 **Feb March 2015**

In the diagram, AB is a chord of the circle with centre O. M is the midpoint of AB. MO is produced to P, where P is a point on the circle. OM = x units, AB = 20 units and $\frac{PM}{OM} = \frac{5}{2}$.



7.1 Write down the length of MB. (1)

7.2 Give a reason why $OM \perp AB$. (1)

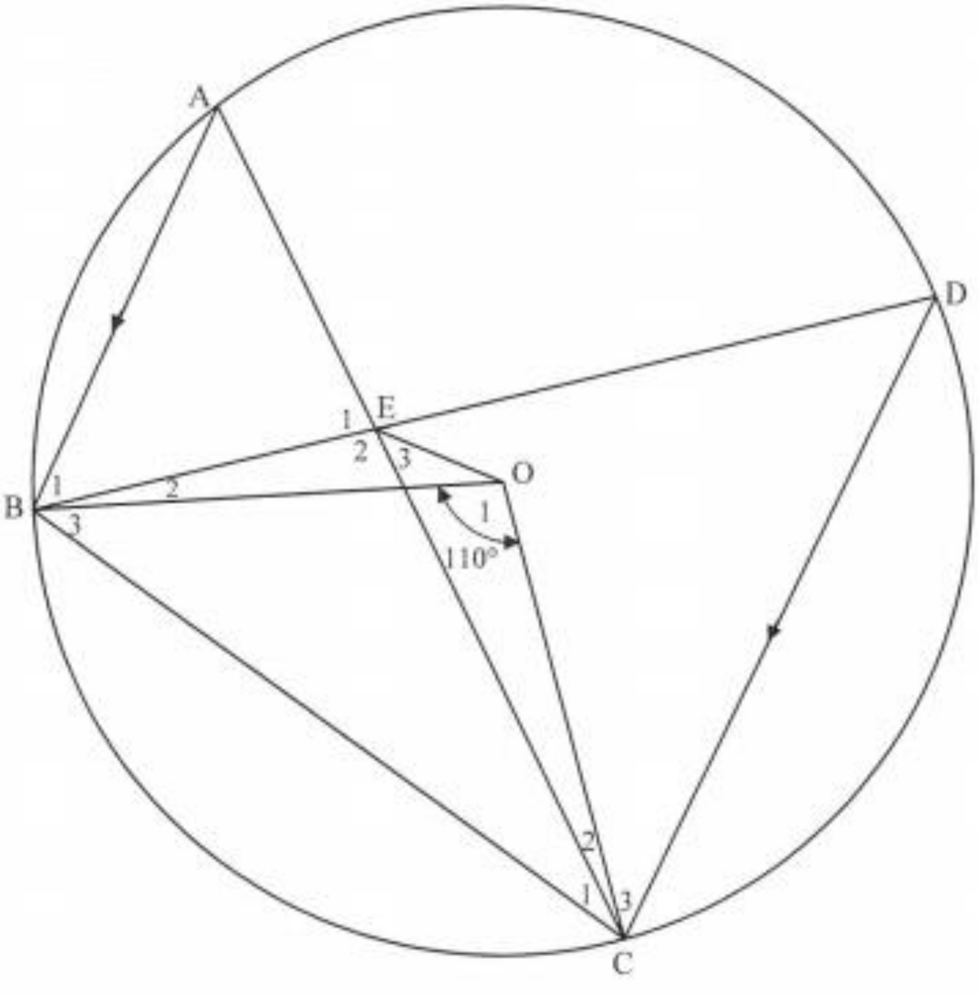
7.3 Show that $OP = \frac{3x}{2}$ units. (2)

7.4 Calculate the value of x. (3)

[7]

Question 8 **Feb March 2015**

In the diagram below, the circle with centre O passes through A, B, C and D.
$AB \parallel DC$ and $B\hat{O}C = 110°$.
The chords AC and BD intersect at E.
EO, BO, CO and BC are joined.



8.1 Calculate the size of the following angles, giving reasons for your answers:

8.1.1 $\hat{D}$ (2)

8.1.2 $\hat{A}$ (2)

8.1.3 $\hat{E}_2$ (4)

8.2 Prove that BEOC is a cyclic quadrilateral. (2)

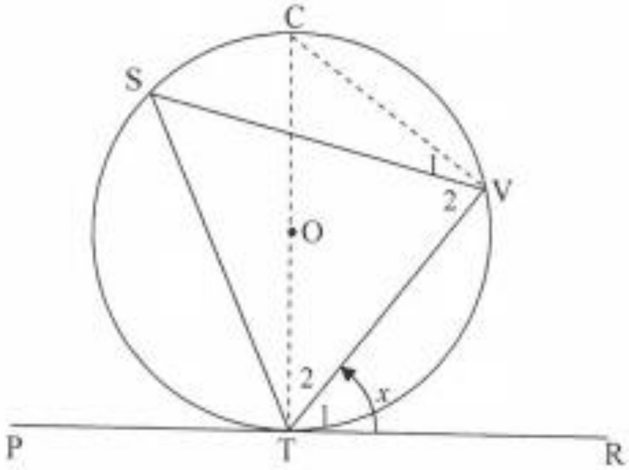
[10]

Question 9 **Feb March 2015**

9.1 Complete the statement of the following theorem:

The exterior angle of a cyclic quadrilateral is equal to ... (1)

9.2 In the diagram below the circle with centre O passes through points S, T and V. PR is a tangent to the circle at T. VS, ST and VT are joined.



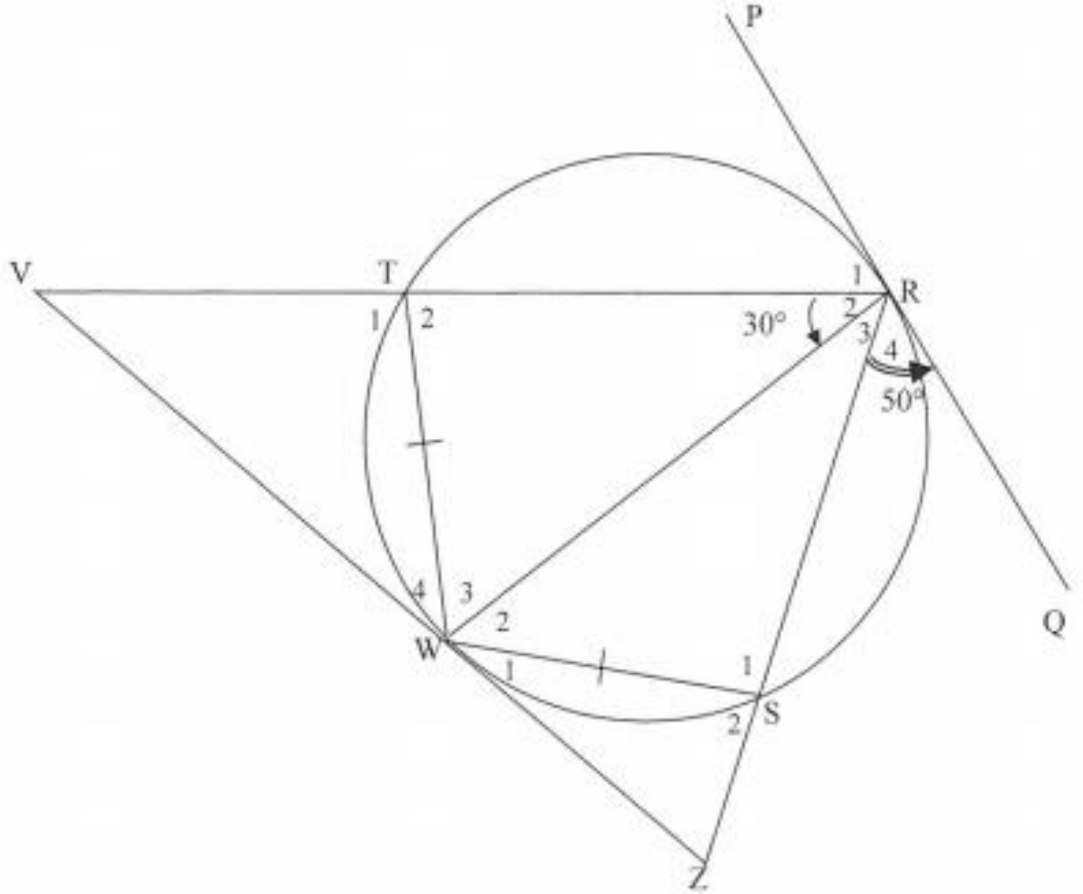
Given below is the partially completed proof of the theorem that states that $V\hat{T}R = \hat{S}$.
Using the above diagram, complete the proof of the theorem on DIAGRAM SHEET 3.

Construction: Draw diameter TC and join CV.

Statement	Reason
Let: $V\hat{T}R = \hat{T}_1 = x$	
$\hat{V}_1 + \hat{V}_2 =$..........	....
$\hat{T}_2 = 90° - x$	....
$\therefore \hat{C} =$	Sum of the angles of a triangle
$\therefore \hat{S} = x$	....
$\therefore V\hat{T}R = \hat{S}$	

(5)

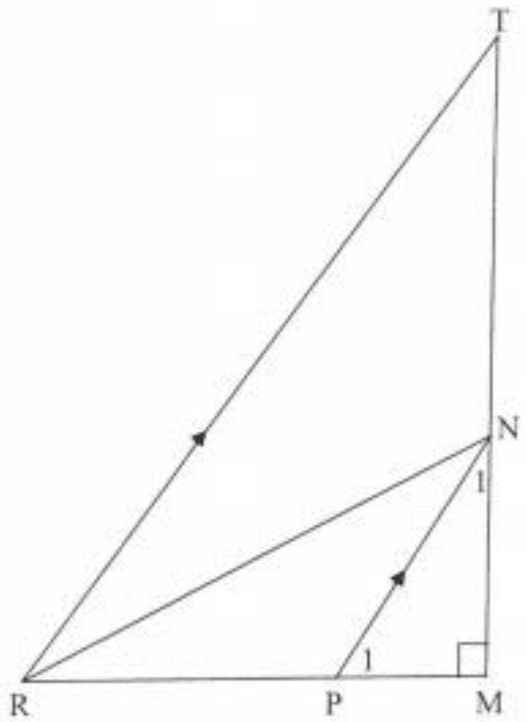
9.3 In the figure, TRSW is a cyclic quadrilateral with TW = WS. RT and RS are produced to meet tangent VWZ at V and Z respectively. PRQ is a tangent to the circle at R. RW is joined. $\hat{R}_2 = 30°$ and $\hat{R}_4 = 50°$.



9.3.1	Give a reason why $\hat{R}_3 = 30°$.	(1)
9.3.2	State, with reasons, TWO other angles equal to 30°.	(3)
9.3.3	Determine, with reasons, the size of:	
	(a) $\hat{S}_2$	(3)
	(b) $\hat{V}$	(4)
9.3.4	Prove that $WR^2 = RV \times RS$.	(5)
		[22]

Question 10 **Feb March 2015**

In ΔTRM, $\hat{M} = 90°$. NP is drawn parallel to TR with N on TM and P on RM. It is further given that RT = 3PN.



10.1 Give reasons for the statements below.
Use DIAGRAM SHEET 5.

	Statement	**Reason**
	In ΔPNM and ΔRTM :	
10.1.1	$\hat{N}_1 = \hat{T}$	....
	$\hat{M}$ is common	
10.1.2	$\therefore \Delta PNM \ ||| \ \Delta RTM$	....

(2)

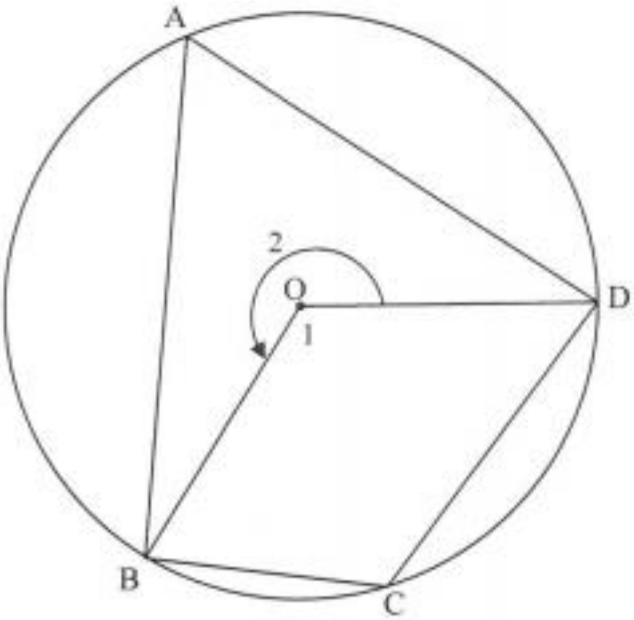
10.2 Prove that $\frac{PM}{RM} = \frac{1}{3}$. (2)

10.3 Show that $RN^2 - PN^2 = 2RP^2$. (4)

[8]

Question 8 **November 2015**

8.1 In the diagram below, cyclic quadrilateral ABCD is drawn in the circle with centre O.

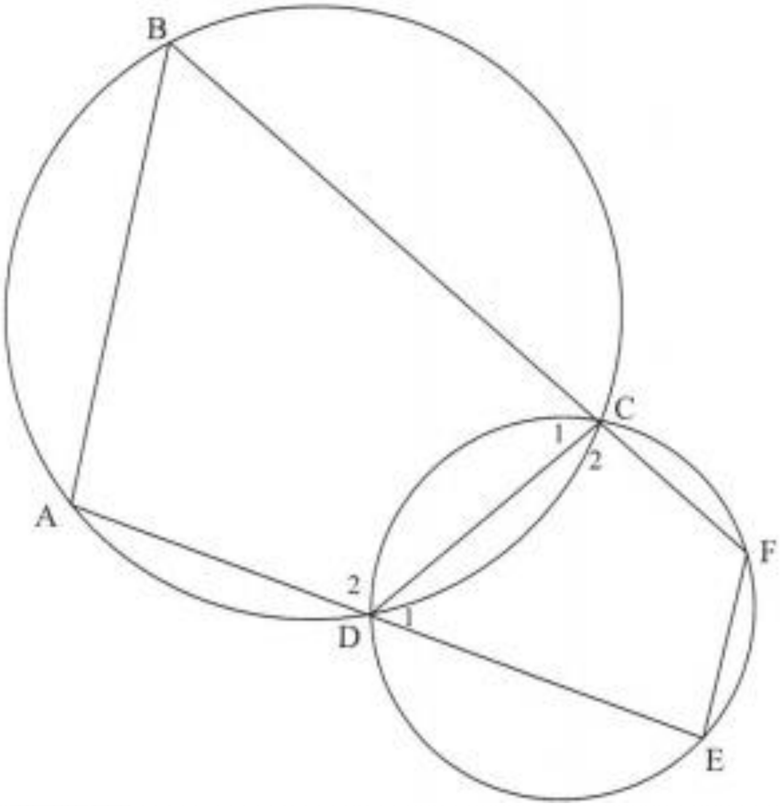


8.1.1 Complete the following statement:

The angle subtended by a chord at the centre of a circle is … the angle subtended by the same chord at the circumference of the circle. (1)

8.1.2 Use QUESTION 8.1.1 to prove that $\hat{A} + \hat{C} = 180°$. (3)

8.2 In the diagram below, CD is a common chord of the two circles. Straight lines ADE and BCF are drawn. Chords AB and EF are drawn.

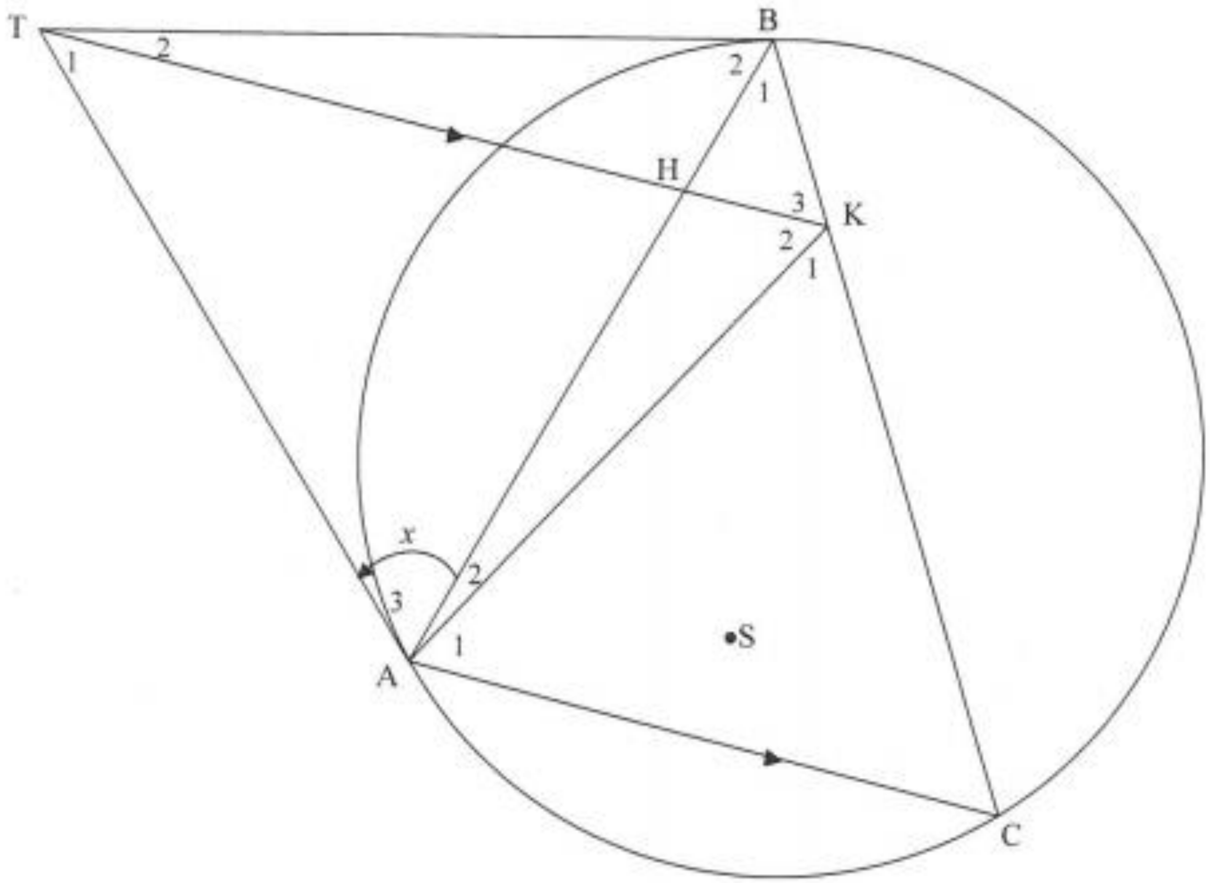


Prove that EF | | AB. (5)

[9]

Question 9 **November 2015**

In the diagram below, ΔABC is drawn in the circle. TA and TB are tangents to the circle. The straight line THK is parallel to AC with H on BA and K on BC. AK is drawn. Let $\hat{A}_3 = x$.



9.1 Prove that $\hat{K}_3 = x$. (4)

9.2 Prove that AKBT is a cyclic quadrilateral. (2)

9.3 Prove that TK bisects $A\hat{K}B$. (4)

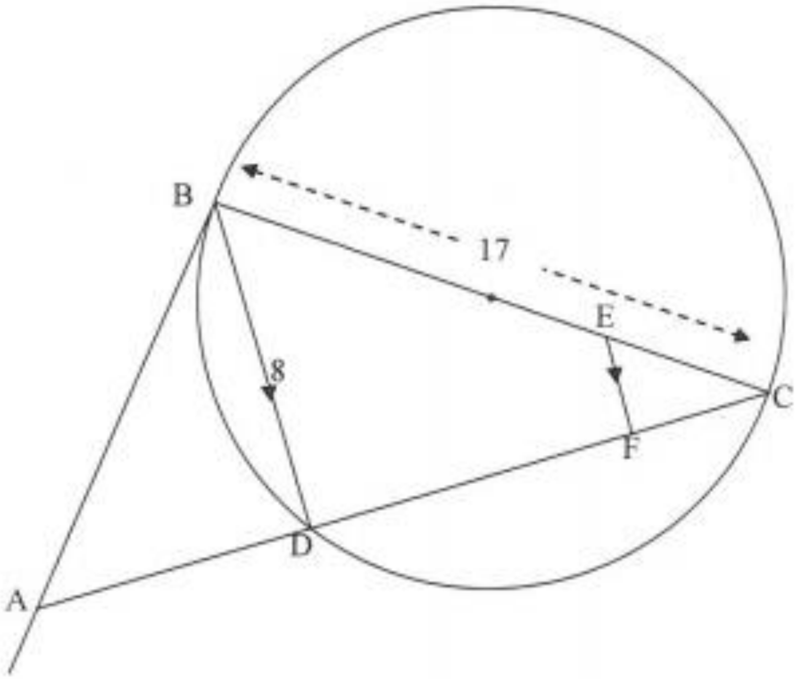
9.4 Prove that TA is a tangent to the circle passing through the points A, K and H. (2)

9.5 S is a point in the circle such that the points A, S, K and B are concyclic. Explain why A, S, B and T are also concyclic. (2)

[14]

Question 10 **November 2015**

In the diagram below, BC = 17 units, where BC is a diameter of the circle. The length of chord BD is 8 units. The tangent at B meets CD produced at A.



10.1	Calculate, with reasons, the length of DC.	(3)
10.2	E is a point on BC such that BE : EC = 3 : 1. EF is parallel to BD with F on DC.	

10.2.1	Calculate, with reasons, the length of CF.	(3)
10.2.2	Prove that $\Delta BAC \mid\mid\mid \Delta FEC$.	(5)
10.2.3	Calculate the length of AC.	(4)
10.2.4	Write down, giving reasons, the radius of the circle passing through points A, B and C.	(2)

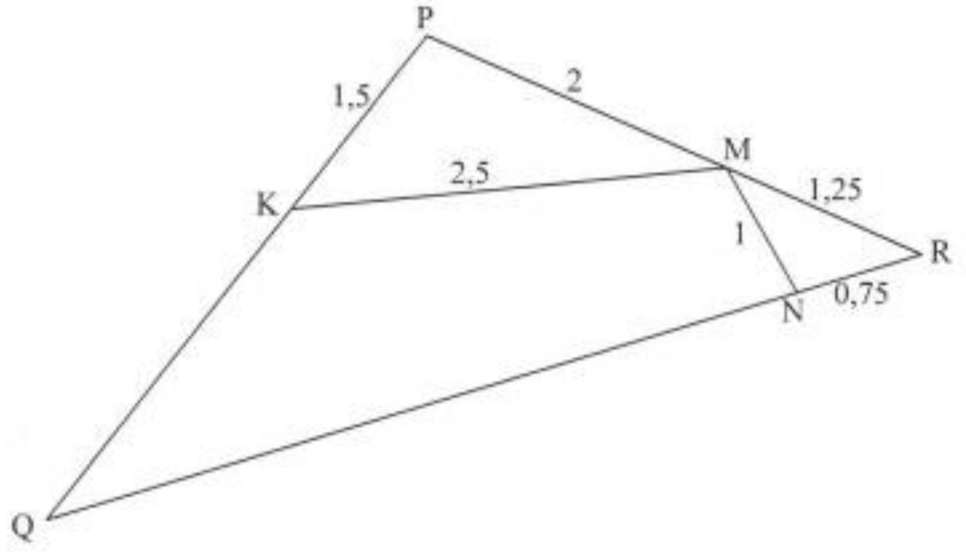
[17]

Question 11 **November 2015**

11.1 Complete the following statement:

If the sides of two triangles are in the same proportion, then the triangles are ... (1)

11.2 In the diagram below, K, M and N respectively are points on sides PQ, PR and QR of ΔPQR. KP = 1,5; PM = 2; KM = 2,5; MN = 1; MR = 1,25 and NR = 0,75.



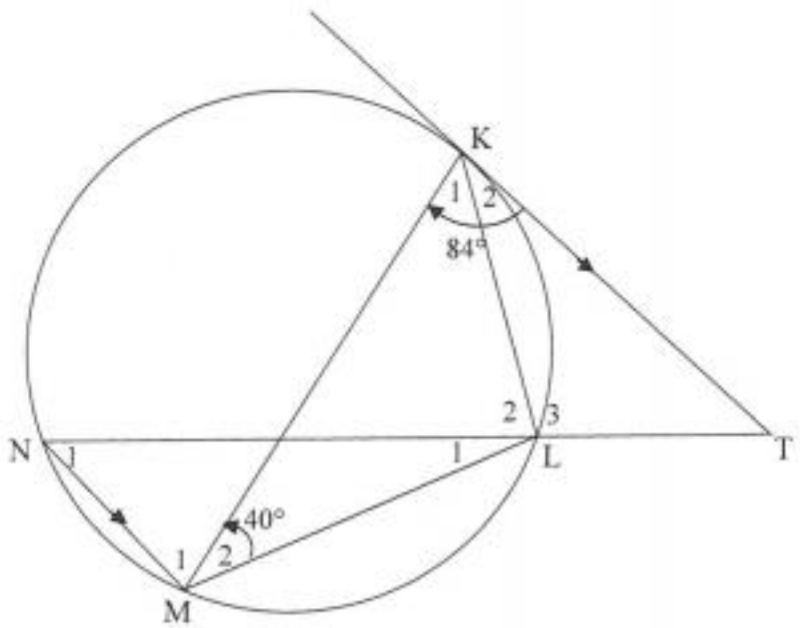
11.2.1 Prove that ΔKPM ||| ΔRNM. (3)

11.2.2 Determine the length of NQ. (6)

[10]

Question 8 **Feb March 2016**

8.1 In the diagram below, tangent KT to the circle at K is parallel to the chord NM. NT cuts the circle at L. ΔKML is drawn. $\hat{M}_2 = 40°$ and $M\hat{K}T = 84°$.



Determine, giving reasons, the size of:

8.1.1 $\hat{K}_2$ (2)

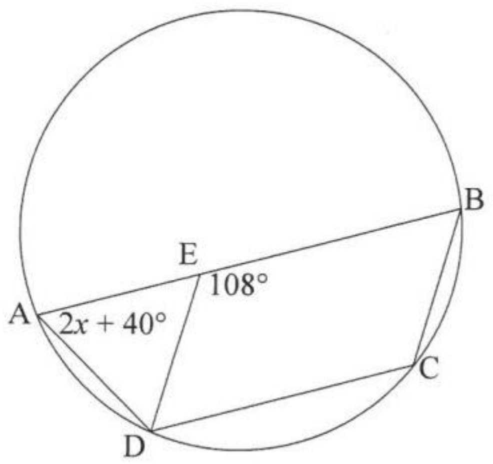
8.1.2 $\hat{N}_1$ (3)

8.1.3 $\hat{T}$ (2)

8.1.4 $\hat{L}_2$ (2)

8.1.5 $\hat{L}_1$ (1)

8.2 In the diagram below, AB and DC are chords of a circle. E is a point on AB such that BCDE is a parallelogram. $D\hat{E}B = 108°$ and $D\hat{A}E = 2x + 40°$.

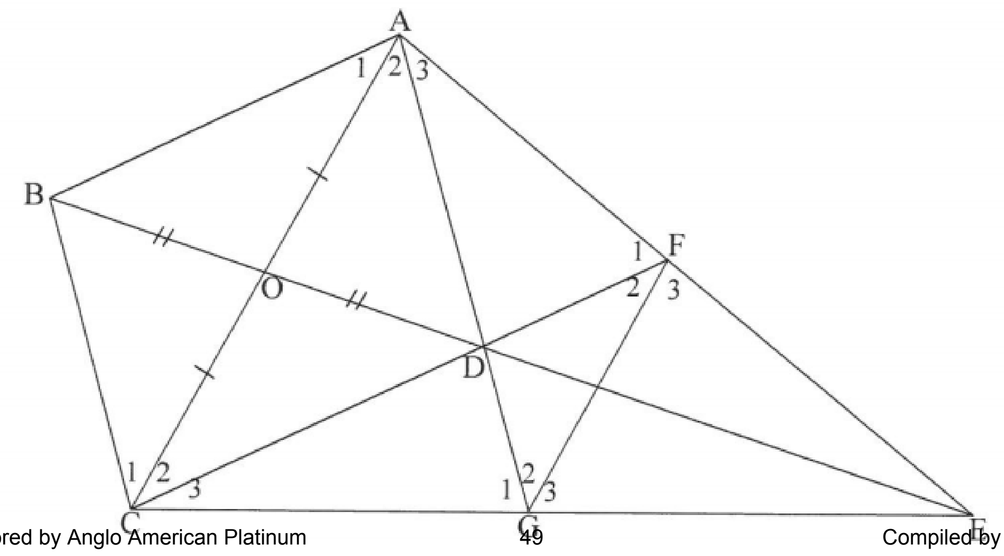


Calculate, giving reasons, the value of x. (5)

[15]

Question 9 **Feb March 2016**

In the diagram below, EO bisects side AC of ΔACE. EDO is produced to B such that BO = OD. AD and CD produced meet EC and EA at G and F respectively.



9.1 Give a reason why ABCD is a parallelogram. (1)

9.2 Write down, with reasons, TWO ratios each equal to $\frac{ED}{DB}$. (4)

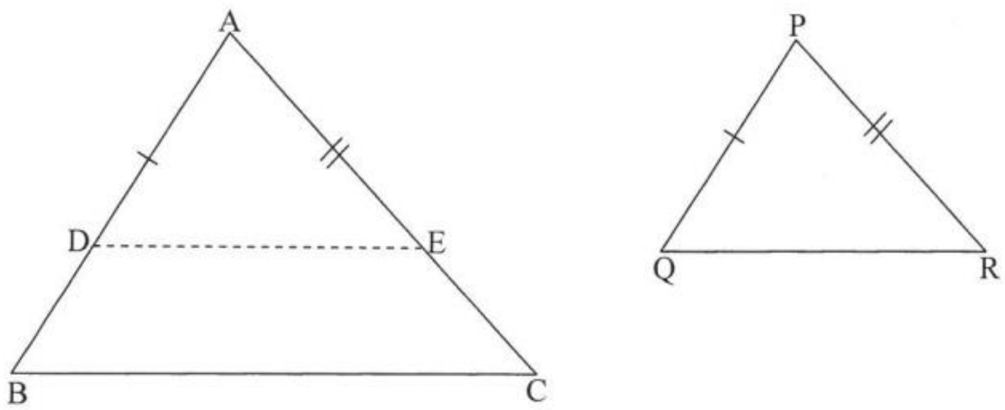
9.3 Prove that $\hat{A}_1 = \hat{F}_2$. (5)

9.4 It is further given that ABCD is a rhombus. Prove that ACGF is a cyclic quadrilateral. (3)

[13]

Question 10 **Feb March 2016**

10.1 In the diagram below, ΔABC and ΔPQR are given with $\hat{A} = \hat{P}$, $\hat{B} = \hat{Q}$ and $\hat{C} = \hat{R}$.



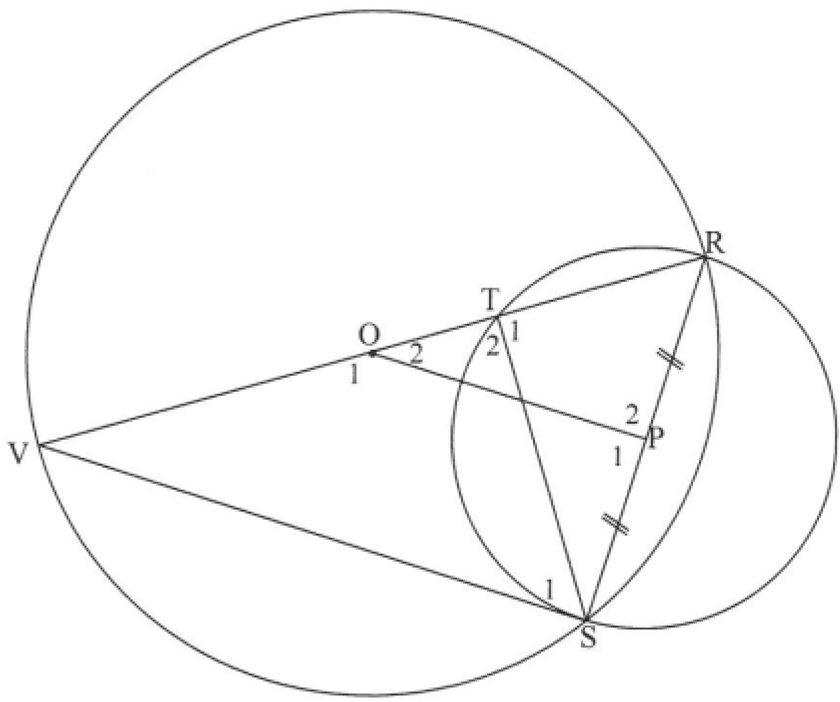
DE is drawn such that AD = PQ and AE = PR.

10.1.1 Prove that $\Delta ADE \equiv \Delta PQR$. (2)

10.1.2 Prove that DE | | BC. (3)

10.1.3 Hence, prove that $\frac{AB}{PQ} = \frac{AC}{PR}$. (2)

10.2 In the diagram below, VR is a diameter of a circle with centre O. S is any point on the circumference. P is the midpoint of RS. The circle with RS as diameter cuts VR at T. ST, OP and SV are drawn.



10.2.1 Why is $OP \perp PS$? (1)

10.2.2 Prove that $\Delta ROP \ ||| \ \Delta RVS$. (4)

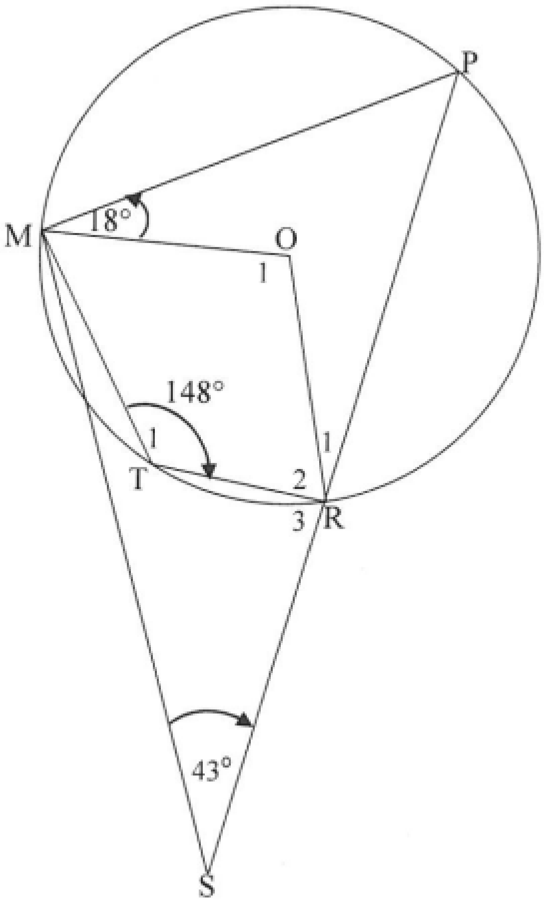
10.2.3 Prove that $\Delta RVS \ ||| \ \Delta RST$. (3)

10.2.4 Prove that $ST^2 = VT \cdot TR$. (6)

[21]

Question 8 **May June 2016**

8.1 In the diagram below, P, M, T and R are points on a circle having centre O. PR produced meets MS at S. Radii OM and OR and the chords MT and TR are drawn. $\hat{T}_1 = 148°$, $P\hat{M}O = 18°$ and $\hat{S} = 43°$



Calculate, with reasons, the size of:

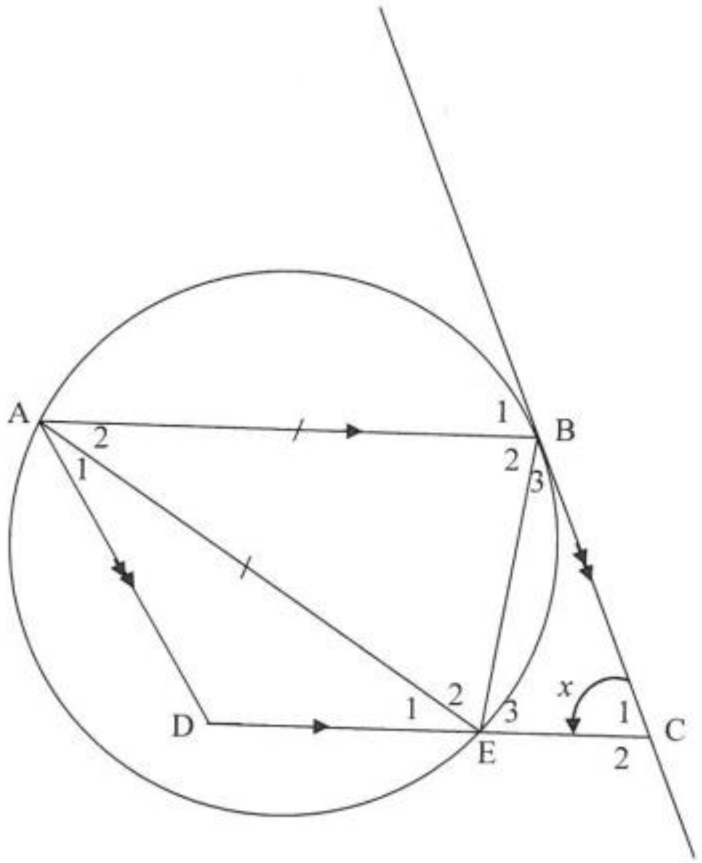
8.1.1 $\hat{P}$ (2)

8.1.2 $\hat{O}_1$ (2)

8.1.3 $O\hat{M}S$ (2)

8.1.4 $\hat{R}_3$ if it is given that $T\hat{M}S = 6°$ (2)

8.2 In the diagram below, the circle passes through A, B and E. ABCD is a parallelogram. BC is a tangent to the circle at B. AE = AB. Let $\hat{C}_1 = x$



8.2.1 Give a reason why $\hat{B}_1 = x$ (1)

8.2.2 Name, with reasons, THREE other angles equal in size to x. (6)

8.2.3 Prove that ABED is a cyclic quadrilateral. (3)

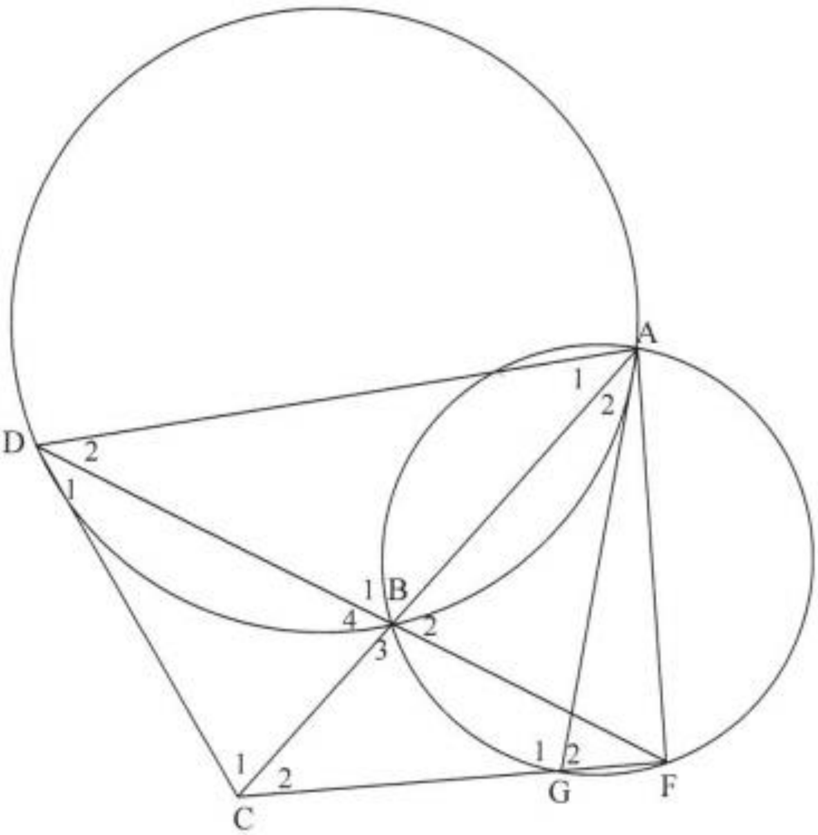
[18]

Question 9 **May June 2016**

9.1 Complete the statement so that it is TRUE:

The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle ... (1)

9.2 In the diagram below, two unequal circles intersect at A and B. AB is produced to C such that CD is a tangent to the circle ABD at D. F and G are points on the smaller circle such that CGF and DBF are straight lines. AD and AG are drawn.



Prove that:

9.2.1 $\hat{B}_4 = \hat{D}_1 + \hat{D}_2$ (4)

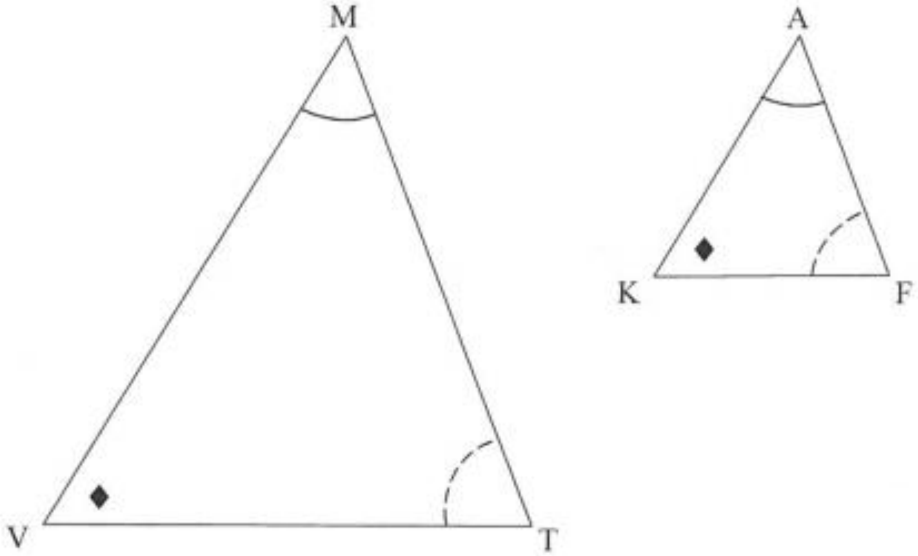
9.2.2 AGCD is a cyclic quadrilateral (4)

9.2.3 DC = CF (4)

[13]

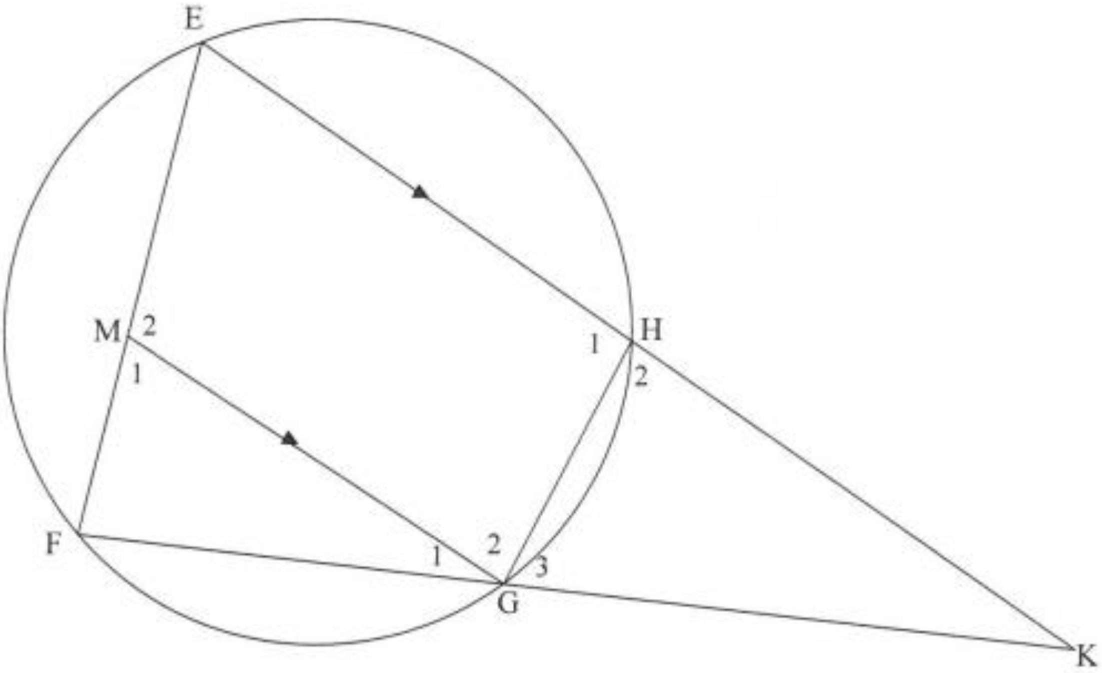
Question 10 **May June 2016**

10.1 In the diagram below, ΔMVT and ΔAKF are drawn such that $\hat{M} = \hat{A}$, $\hat{V} = \hat{K}$ and $\hat{T} = \hat{F}$



Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, that is $\frac{MV}{AK} = \frac{MT}{AF}$ (7)

10.2 In the diagram below, cyclic quadrilateral EFGH is drawn. Chord EH produced and chord FG produced meet at K. M is a point on EF such that MG | | EK. Also KG = EF



10.2.1 Prove that:

(a) ΔKGH | | | ΔKEF (4)

(b) $EF^2 = KE \cdot GH$ (2)

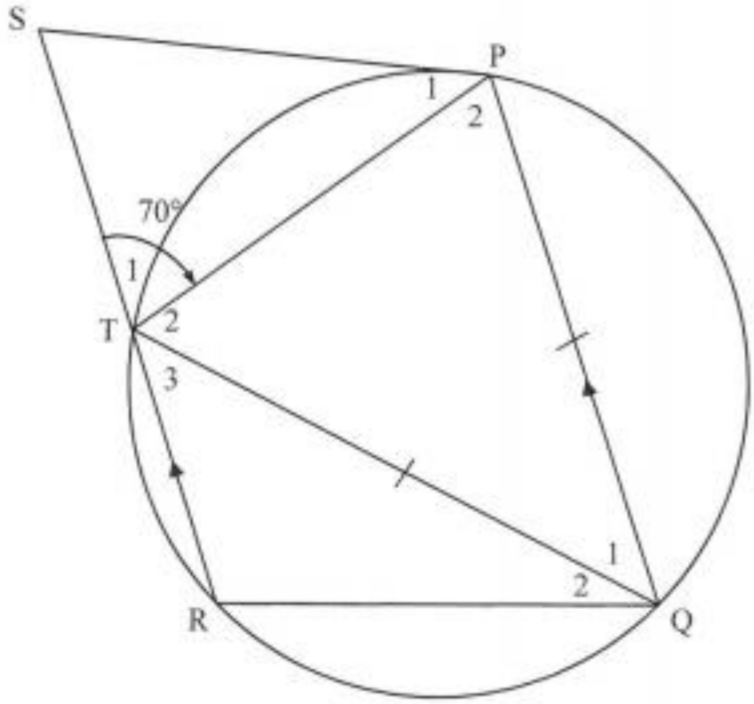
(c) $KG^2 = EM \cdot KF$ (3)

10.2.2 If it is given that KE = 20 units, KF = 16 units and GH = 4 units, calculate the length of EM. (3)

[19]

Question 8 **November 2016**

8.1 In the diagram below PQRT is a cyclic quadrilateral having RT || QP. The tangent at P meets RT produced at S. QP = QT and $P\hat{T}S = 70°$.



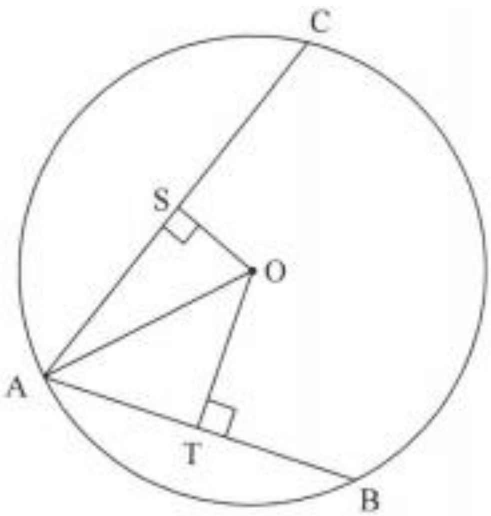
8.1.1 Give a reason why $\hat{P}_2 = 70°$. (1)

8.1.2 Calculate, with reasons, the size of:

(a) $\hat{Q}_1$ (3)

(b) $\hat{P}_1$ (2)

8.2 A, B and C are points on the circle having centre O. S and T are points on AC and AB respectively such that $OS \perp AC$ and $OT \perp AB$. AB = 40 and AC = 48.



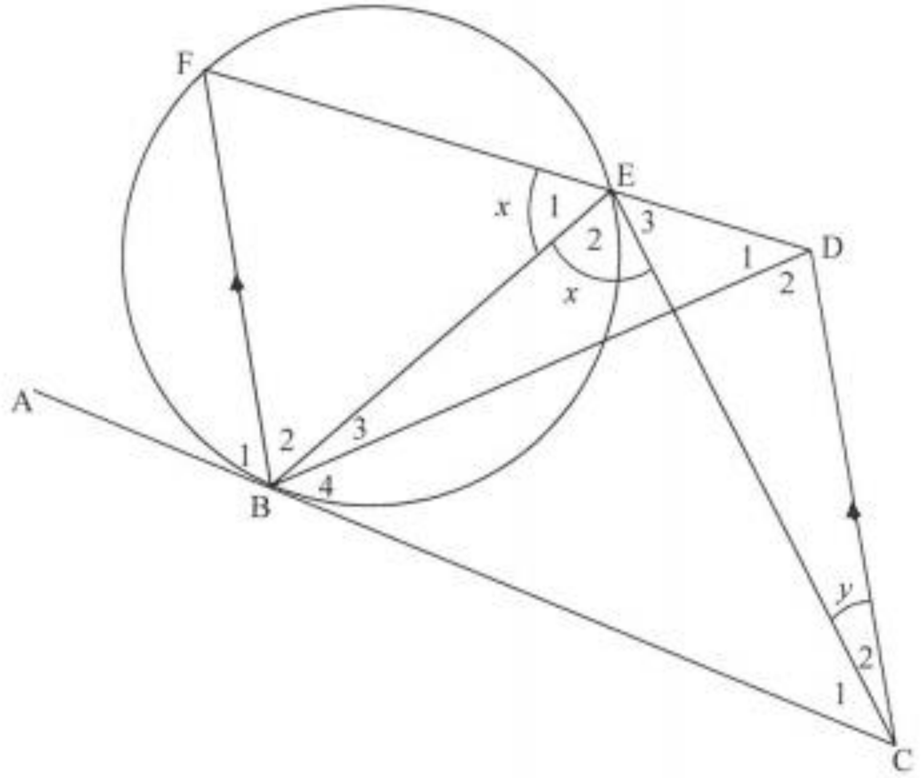
8.2.1 Calculate AT. (1)

8.2.2 If $OS = \frac{7}{15}OT$, calculate the radius OA of the circle. (5)

[12]

Question 9 **November 2016**

ABC is a tangent to the circle BFE at B. From C a straight line is drawn parallel to BF to meet FE produced at D. EC and BD are drawn. $\hat{E}_1 = \hat{E}_2 = x$ and $\hat{C}_2 = y$.



9.1 Give a reason why EACH of the following is TRUE:

9.1.1 $\hat{B}_1 = x$ (1)

9.1.2 $B\hat{C}D = \hat{B}_1$ (1)

9.2 Prove that BCDE is a cyclic quadrilateral. (2)

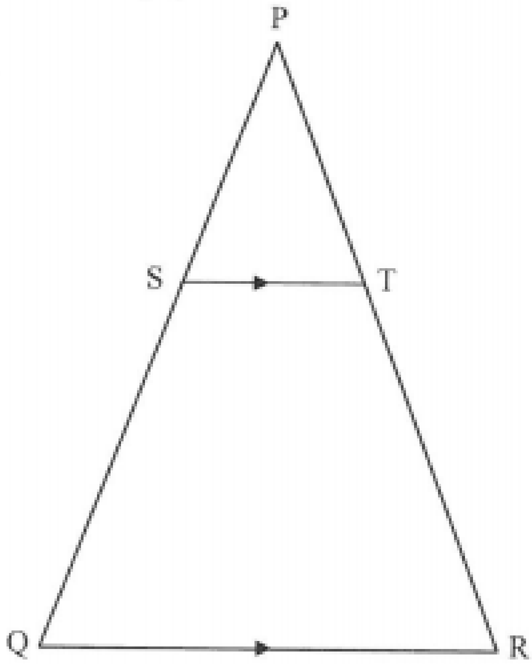
9.3 Which TWO other angles are each equal to x? (2)

9.4 Prove that $\hat{B}_2 = \hat{C}_1$. (3)

[9]

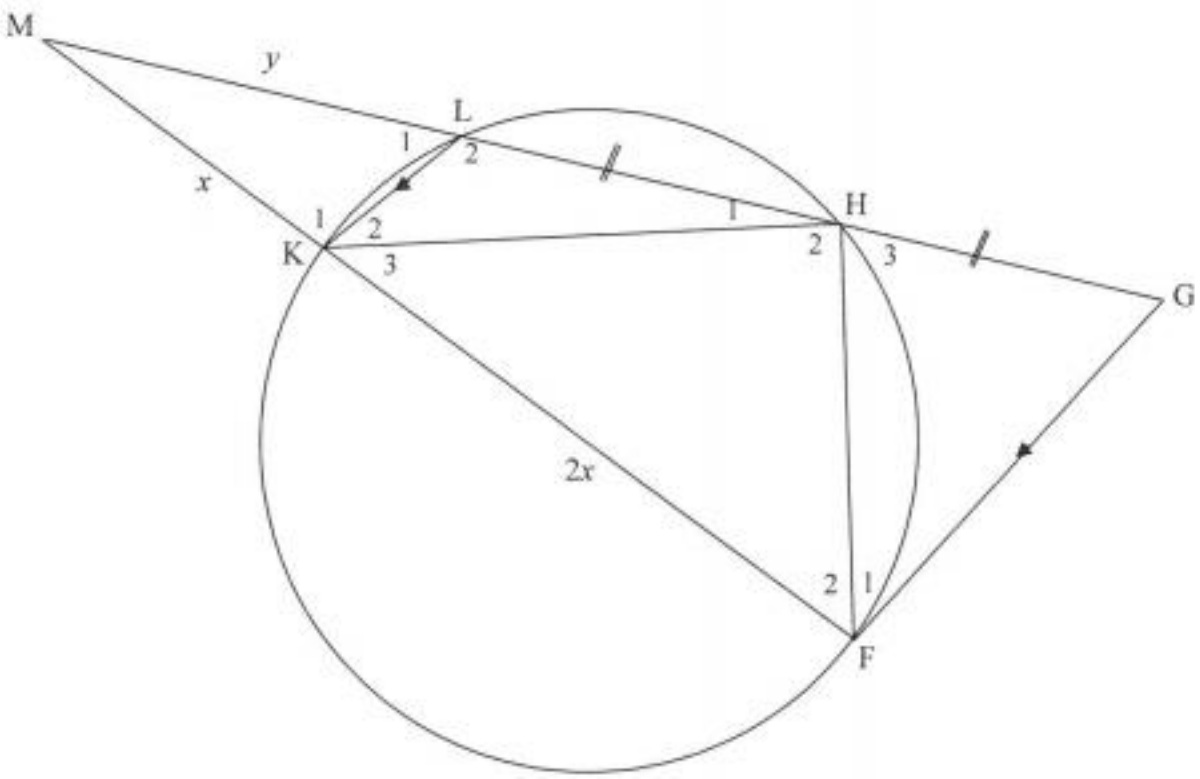
Question 10 **November 2016**

10.1 In the diagram ΔPQR is drawn. S and T are points on sides PQ and PR respectively such that $ST \parallel QR$.



Prove the theorem which states that $\frac{PS}{SQ} = \frac{PT}{TR}$. (6)

10.2 In the diagram HLKF is a cyclic quadrilateral. The chords HL and FK are produced to meet at M. The line through F parallel to KL meets MH produced at G. $MK = x$, $KF = 2x$, $ML = y$ and $LH = HG$.



10.2.1 Give a reason why $G\hat{F}M = L\hat{K}M$. (1)

10.2.2 Prove that:

(a) $GH = y$ (3)

(b) $\Delta MFH \,|||\, \Delta MGF$ (5)

(c) $\frac{GF}{FH} = \frac{3x}{2y}$ (2)

10.2.3 Show that $\frac{y}{x} = \sqrt{\frac{3}{2}}$ (3)

[20]